How Do Humans Write Code? Large Models Do It the Same Way Too

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Abstract

Program-of-Thought (PoT) replaces natural language-based Chain-of-Thought (CoT) as the most popular method in Large Language Models (LLMs) mathematical reasoning tasks by utilizing external tool calls to circumvent computational errors. However, our evaluation of the GPT-4 and Llama series reveals that using PoT introduces more reasoning errors, such as incorrect formulas or flawed logic, compared to CoT. To address this issue, we propose Human-Think Language (HTL), which leverages a suite of strategies that help integrate PoT and CoT, encompassing: (1) a new generation paradigm that uses full CoT reasoning to control code generation. (2) Focus Attention, that directs model attention to the CoT reasoning during PoT to generate more logical code. (3) reinforcement learning that utilizes the accuracy of both CoT and PoT responses as rewards to prevent repetitive reasoning steps in LLMs when solving difficult math problems. Our method achieves an average improvement of 6.5% on the Llama-Base model and 4.3% on the Mistral-Base model across 8 mathematical calculation datasets. It also shows significant effectiveness on five out-of-domain datasets by controlling the model's information flow, exhibiting strong transferability. Additionally, HTL shows the most significant improvement in nonmathematical natural language inference task, contributing to a unified reasoning task framework¹.

1 Introduction

Solving Mathematical reasoning problems is a significant challenge for current LLMs (Madaan et al., 2022; OpenAI et al., 2023). This task requires interpreting information, identifying relevant mathematical concepts, and formulating equations to solve the problems (Ahn et al., 2024). Due to computational errors in LLMs (Wei et al., 2023; Gao

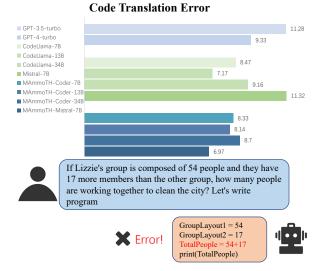


Figure 1: The top section of the chart represents the average CTE for each model across 5 datasets. Below is a real example from the Asdiv dataset using the MAmmoTH-Mistral-7B model, which achieved an accuracy of 93.9% on this dataset. The proportion of CTE remains high across various models, and these errors do not diminish with an increase in model parameters.

et al., 2023), using CoT (Wang et al., 2023b; Wei et al., 2022; Chen et al., 2024) solely implemented in natural language can lead to calculation mistakes (Lewkowycz et al., 2022; Wei et al., 2023; Gao et al., 2023). The most common practice currently is to use PoT (Chen et al., 2023) for handling mathematical reasoning tasks, by guiding the large model to write the code that is then computed using tool calls.

However, we made a surprising discovery recently: when a problem is phrased in a manner closer to verbal scenarios (for example, the question is "One apple costs three dollars, how much for three apples?" instead of " $3 \times 3=$?"), PoT tends to make more reasoning errors or text comprehension mistakes, but this phenomenon is almost nonexistent in CoT. For such problems, CoT can correctly reason out the answer, whereas PoT makes

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¹Code is available at: https://github.com/seamoke/Human-Think-Language

mistakes. We refer to this type of error as **Code Translation Error (CTE)**. We report the percentage of CTE on five datasets with multiple types of models, the results illustrated in Figure 1. This error is due to the amount of training data for natural language far exceeding that for code. In the scope of CodeLlama's pretraining data, which includes 500 billion code tokens, this represents a small fraction compared to the 2 trillion natural language tokens used in the Llama-2 model (Rozière et al., 2023; Hugo Touvron, 2023). Natural language is more suitable for semantic analysis, planning, and abstract reasoning than code (Gou et al., 2023b).

Existing work also finds the advantage of Natural language, but they have not effectively utilized the reasoning capabilities of natural language. Current research focuses on the following approaches to integrate natural language to enhance the precision of code: (1) Using natural language prompts to guide the model in writing code (Gao et al., 2023; Toshniwal et al., 2024; Wang et al., 2023a): write a brief step in natural language before generating code. (2) Employing methods like self-correction and hybrid approaches to generate answers in multiple stages (Gou et al., 2023b; Yue et al., 2023; Gou et al., 2023a). (3) Utilizing prompts like "rethink question" (Deng et al., 2023) to have the model first paraphrase the question, thereby avoiding comprehension errors. However, existing methods fall short in two main aspects: First, using few natural language steps or simple paraphrasing methods alone is insufficient for effectively controlling code generation; a more comprehensive natural language reasoning process is necessary to generate more reliable code. Secondly, reasoning within LLMs is not always faithful (Lanham et al., 2023; Bao et al., 2024; Turpin et al., 2023). Frequently, the final answers seem to be derived directly from the questions themselves rather than aligning with the reasoning process. Consequently, even correct reasoning can lead to incorrect answers.

To more effectively utilize natural language reasoning to enhance PoT, we propose **Human-Think Language (HTL)**: A novel information-controlbased approach to utilize complete CoT reasoning steps to control PoT generation. HTL is inspired by the way humans write code. Humans consider the entire reasoning process using natural language, and the code can fully rely on natural language reasoning. On the right side of Figure 2, we highlight the parallels between our integrated model and the human approach to solving mathematical problems. Compared to previous works, our framework offers a strong capacity for aligning calculation with reasoning by integrating CoT and PoT. We design Focus Attention mechanism that, during code generation, concentrates solely on information from CoT to promote the chain reasoning better, thereby biasing the answer to be more faithful to CoT. On the other hand, using complete CoT reasoning tends to lead LLMs to use mathematical induction to enumerate reasoning steps verbosely, which results in repetitive generation. We incorporate the error assessment function based on PPO (Schulman et al., 2017), leveraging reinforcement learning to penalize repetitive generation. We conduct experiments based on CodeLlama-7B and Mistral-7B and achieve outstanding results on eight datasets using only self-distillation data.

In summary, our contributions are as follows:

(1) We are the first to conduct a detailed evaluation of current closed-source models, open-source base models, and specialized models. We highlight the shortcomings of PoT and propose that using full natural language reasoning to enhance PoT performance is essential.

(2) We propose an advanced model named HTL, which utilizes the complete reasoning process of CoT to enhance PoT. HTL incorporates a novel Focus Attention that approximates chain reasoning, complemented by an error assessment function designed to prevent repetitive generation.

(3) We evaluate our work on eight mathematical reasoning datasets, and our experimental results demonstrate that our method achieves outstanding results. HTL shows significant effectiveness in indomain datasets, out-of-domain datasets, and natural language inference task, demonstrating strong usability and potential.

2 Method

The design of HTL is divided into three parts: reasoning format, Focus Attention, and error assessment function based PPO.

2.1 Reasoning Format

In Human-Think Language (HTL), we introduce a new reasoning paradigm that uses full CoT reasoning to control the process of PoT, as shown in Figure 3. The CoT approach, despite sometimes producing erroneous computational outcomes, follows a generally correct skeleton (Wang et al., 2024)

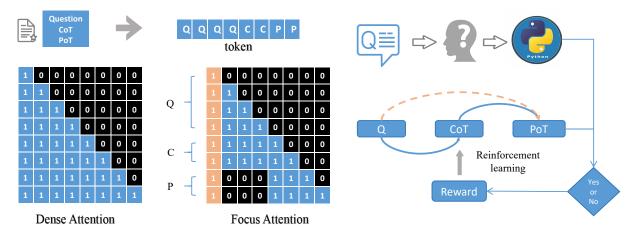


Figure 2: Dense Attention refers to traditional Attention, while Focus Attention is our approach. In the orange column on the left, the first four tokens share a consistent mask state of 1. On the right side of the figure, there is a comparison between human and LLMs in solving mathematical problems.

for reasoning. By adapting code translation after the CoT reasoning path, the PoT can inherit the reasoning skeleton of CoT while circumventing its computational errors. This effectively combines the advantages of both approaches.

Human-Think Language
Question
If $\frac{2}{5} = \frac{A}{60} = \frac{60}{B}$, what is $A + B$?
Generated Solution
We have $\frac{2}{5} = \frac{60}{B} \Rightarrow B = \frac{60}{2} = 30.$
We also have $\frac{60}{A} = \frac{2}{5} \Rightarrow A = 60$.
Thus $A + B = 60 + 30 = 90$. The answer is 90.
<pre># Python program to calculate A + B from sympy import symbols, solve # Define the variables</pre>
A, B = symbols('A_B') # Equation 1: 2/5 = A/60
eq1 = solve (2/5 - A/60, A)[0]
Equation 2: 60/B = 2/5 eq2 = solve (60/B - 2/5, B)[0]
<pre># Calculate A + B result = eq1 + eq2</pre>
print (result)

Figure 3: Demonstrating a successful example for HTL: Although the CoT's answer may contain many calculation errors (in red), its reasoning skeleton is correct. HTL enables PoT to follow CoT's reasoning steps to arrive at the correct result.

2.2 Focus Attention

Attention Design In our work, we use a local attention (Beltagy et al., 2020) mechanism to control the information flow during training (Figure 2). We divide the text of a mathematical problem into three parts: Q (question), C (CoT), and P (PoT). The objective in generating the PoT reasoning is for the model to rely solely on information from the CoT reasoning, not on the question. However, a recent study by (Xiao et al., 2023) introduced the concept of attention-sink, showing that the initial tokens of a sequence attract a significant portion of attention. Therefore, while the Focus Attention mechanism masks the information from Q and focuses solely on C during PoT generation, it preserves the initial tokens in the sequence to prevent the loss of substantial information. Echoing the findings of (Xiao et al., 2023), we include the first four tokens in the PoT information generation process. This results in the following modified formula for the casual mask matrix:

$$M_{ij} = \begin{cases} 0 & j \le i \land (j \in \{0, 1, 2, 3\} \lor j \in \mathbf{C}) \\ -\infty & \text{otherwise} \end{cases}$$

(1)

Ultimately, the contextualized representation X^l of at *l*-th attention layer can be formulated as:

$$A^{l} = \text{Softmax}\left(\frac{X^{l-1}W_{Q}^{l}\left(X^{l-1}W_{K}^{l}\right)^{T}}{\sqrt{d/N}} + M\right)$$
$$X^{l} = A^{l}\left(X^{l-1}W_{V}^{l}\right)$$
(2)

 W_Q , W_K , and W_V are intermediate matrix representations in the attention mechanism. X is the

hidden vector representation of the sequence.

Adaptive Training Strategy To align with the dense causal matrix used for both pretraining and inference, which is inconsistent with our Focus Attention, we introduce a novel training approach: during both the initial and final phases of training, we do not explicitly mask any tokens besides the causal mask, thereby ensuring alignment with the pretraining stage and the inference stage. In the middle of the training process, we incorporate a **mask coverage function**, which is a quadratic function and calculates a proportion of entries to be randomly masked based on the number of training steps, allowing the mask to transition between Dense Attention and Focus Attention:

$$\lambda_{masked} = min(1, -\alpha(\rho_{step} - \frac{1}{2})^2 + \beta) \quad (3)$$

where λ is the percentage of masked entries, and ρ_{step} is the current training step divided by the total steps. Then, we randomly select the parts to mask in the mask matrix based on the values of the mask coverage function. It is noteworthy that during the **inference phase**, HTL utilizes the traditional causal mask.

2.3 Error Assessment Function Based PPO

In reinforcement-learning stage, We employ PPO with a clipped objective algorithm for training. Following (Ziegler et al., 2020), the value model V_{ϕ} is constructed by appending a linear value head on top of the last hidden states of the policy model. For the reward model, we replace it with error assessment function. At the terminal state, we use a reward function to score based on the correctness of the generated answer. All other states are assigned a value of 0. We categorize the reasons for their errors and provide more fine-grained feedback scores for the model based on the answers from CoT and PoT. The error assessment function is as follows:

$$f_r = \begin{cases} 1, & \operatorname{CoT} = \boldsymbol{y}, \operatorname{PoT} = \boldsymbol{y} \\ 0.6, & \operatorname{CoT} \neq \boldsymbol{y}, \operatorname{PoT} = \boldsymbol{y} \\ 0.3, & \operatorname{CoT} = \boldsymbol{y}, \operatorname{PoT} \neq \boldsymbol{y} \\ 0.1, & \operatorname{CoT} = \operatorname{null} \operatorname{or} \operatorname{PoT} = \operatorname{null} \\ 0, & \operatorname{CoT} = \operatorname{null} \operatorname{and} \operatorname{PoT} \neq \operatorname{null} \end{cases}$$
(4)

In cases where the model cannot produce an answer, we consider it as model-level error and apply the harshest penalty. If the CoT is correct and PoT is incorrect, we consider it as code translation error. In cases where only CoT is incorrect but PoT is correct, we view it as solely a calculation error and put a slight penalty. Such a partial reward can help reduce the negative effect of learning from sparse rewards. Furthermore, in line with (Zheng et al., 2023), our total reward encompasses both the reward function score and the Kullback-Leibler (KL) divergence (Kullback and Leibler, 1951) between the learned RL policy and the initial policy. For the remaining details on PPO, we refer to (Luong et al., 2024).

3 Experiment

3.1 Baseline

Our work is based on the MAmmoTH model (Yue et al., 2023), which achieves outstanding performance in open-source LLM mathematical reasoning by Hybrid Instruction Tuning from a mixture of CoT and PoT for training². MAmmoTH has two bases: CodeLlama-7B (Rozière et al., 2023) and Mistral-7B (Jiang et al., 2023). We compare the following methods:

PoT/PAL (Gao et al., 2023) uses the LLM to read natural language problems and generate programs as intermediate reasoning steps, but offloads the solution step to a runtime such as a Python interpreter. PAL is a more refined version of PoT, with each line of code accompanied by a comment.

Hybrid Approach (Yue et al., 2023) first performs a PoT execution. If the program has any syntax errors, the answer is obtained through CoT.

Rephrase-and-Respond (RAR) (Deng et al., 2023) enables LLMs to rephrase and expand on the questions posed by humans, followed by the responses within a single prompt. This approach serves as a simple yet effective method for improving performance with a two-stage generation process.

Other Models such as the strong closed-source model GPT-4 and two 7B open-source models that integrate natural language and code: ToRA (Gou et al., 2023b)³ and MathCoder (Wang et al., 2023a).

²OpenMathInstruct (Toshniwal et al., 2024) primarily consists of PoT data, which is not suitable for our experimental comparisons.

³ToRA perform better on the GSM8K and MATH datasets, but ToRA has not open-sourced its datasets.

3.2 Experimental Setting

For reinforcement learning, we set a uniform number of 10 epochs, with the KL coefficient set to 0.01. The learning rates for CodeLlama-Base and Mistral are 1e-5 and 2e-6, respectively. For the SFT stage, the specific parameters are as shown in Table 1. For the mask coverage function, we set α to -11.0 and β to 1.76. For fair comparison with sota, we follow the standard evaluation protocols⁴.

Model	Epoch	Batch Size	lr
CodeLlama-Base	2	64	2e-5
Mistral-Base	2	64	5e-6

Table 1: Details of training hyperparameters for finetuning the different base models. Batch size = the batch size per GPU * the number of GPUs * gradient accumulation steps.

3.3 Dataset

Training Dataset We use hybrid data from the training set of the MAmmoTH model. We first run the fine-tuned MAmmoTH model to generate both CoT and PoT answers for them. We then convert the data into the format of Q, C, and P and discard any incomplete data. In the end, we extract 36,000 examples, with 18,000 coming from GSM8K (Cobbe et al., 2021), 3,000 from NumGLUE (Mishra et al., 2022), and 15,000 from MATH. Using training data from self-distillation can mitigate the effect of performance differences among models with different bases.

Test Dataset Our experiments test on eight datasets: GSM8K, NumGLUE, Math, SimulEq (Koncel-Kedziorski et al., 2016), Deep-Mind (Saxton et al., 2019), SVAMP (Patel et al., 2021), MAWPS (Koncel-Kedziorski et al., 2016) and Asdiv (Miao et al., 2020). These eight datasets have varying levels of difficulty and length, comprehensively reflecting the model's mathematical computational capabilities. Meanwhile, GSM8K, NumGLUE and MATH are in-domain datasets, whereas others are out-of-domain datasets.

3.4 Main Results

The main results are shown in Table 2. Our method clearly surpasses other existing methods, achieving state-of-the-art (SOTA) across multiple datasets.

Most noticeably, our method exhibits a significant improvement on the NumGLUE dataset, because the dataset contains a large amount of natural language inference, which is unsuitable for direct PoT. On average, HTL improved 5% performance for Llama-Base and 4% for Mistral-Base. We will discuss some detailed findings below.

For the experiments with PoT (4-shot) and PAL (4-shot), since the current PoT already uses meaningful variable names in the code, adding additional comments by PAL results in a very slight improvement. For the experiment with RAR, while it can reduce some misunderstandings the model has about the problem, it cannot prevent incorrect reasoning. For the hybrid approach that switches to CoT upon errors in code execution, while it has a 9.8% improvement on NumGLUE over vanilla PoT, HTL achieves an 8.7% improvement over it by establishing a closer connection between CoT and PoT in a unified one-stage generation process. Compared to proprietary models, GPT-4 still exhibits strong performance, widening the gap with open-source models. ToRA and MathCoder use data generated from GSM8K and MATH datasets. Our performance on these two datasets is not as good as ToRA's, but we have excellent generalizability, showing significant improvements on outof-domain datasets. Our method of controlling information flow exhibits strong transferability because it directly changes how the model acquires information, making its effectiveness not limited to in-domain datasets.

We also conduct experiments for a two-stage version of HTL and observed no consistent performance gain of vanilla one-stage generation over it. This shows that the performance gain mainly comes from Focus Attention and Reinforcement Learning we designed for the one-stage paradigm.

3.5 Ablation Study

We conducted ablation experiments on all datasets to investigate the contribution of each key component or strategy of our proposed method. Ablation experiments include two aspects: method ablation and data ablation.

Method Ablation The ablation tests include w/o Focus Attention and w/o reinforcement learning. Focus Attention is a powerful enhancement for HTL, directly improving performance by an average of 2%. It effectively helps the model focus on useful information. For reinforcement learn-

⁴https://github.com/TIGER-AI-Lab/MAmmoTH

Model	Method	GSM8K	NumGLUE	MATH	SimulEq	DeepMind	SVAMP	MAWPS	ASDiv	Avg
GPT-4	РоТ	97.0*	-	69.7 [*]	-	-	94.8*	97.7*	92.6*	-
MathCoder	Mix	67.8^{*}	-	30.2^{*}	49.6^{*}	-	70.7^*	-	-	-
ToRA	Tool-integrate	72.6^{*}	46.2	44.6^{*}	48.5	55.9	70.4^*	91.3 [*]	78.7^*	63.53
				CodeL	lama					
MAmmoTH	PoT(4-shot)	58.6	52.5	31.7	37.4	52.0	72.1	91.7	68.2	58.05
MAmmoTH	PAL(4-shot)	58.8	53.3	30.9	38.3	52.3	72.0	91.7	70.3	58.45
MAmmoTH	РоТ	58.9	56.6	32.8	44.1	53.7	70.7	91.9	69.3	59.75
MAmmoTH	Hybrid	59.4	66.4	33.4	45.9	59.8	71.4	92.0	69.3	62.20
MAmmoTH	RAR	61.2	57.3	32.7	45.3	61.2	72.1	91.6	72.2	61.69
HTL	Two-stage	59.6	70.6	32.2	50.5	61.2	70.3	92.3	70.9	63.45
HTL	-	61.7	63.0	33.9	48.6	61.1	71.5	92.8	71.6	63.03
	+focus	63.9	74.1	34.1	50.9	63.1	72.3	95.0	74.0	65.96
	+RL	68. 7	66.9	34.5	45.8	61.3	76.1	92.3	72.9	65.81
	+focus+RL	65.7	75.1	34.9	50.8	62.8	74.4	94.2	73.1	66.27
				Mist	ral					
MAmmoTH	РоТ	74.5	73.9	37.1	48.2	55.8	80.5	93.9	74.7	67.33
MAmmoTH	Hybrid	75.0	73.9	39.7	50.3	61.1	80.6	93.9	74.7	68.65
MAmmoTH	RAR	76.3	74.7	37.3	49.3	54.3	80.3	93.7	74.8	67.59
HTL	-	74.7	76.3	38.5	51.6	62.9	81.2	93.7	76.2	69.38
	+focus	77.9	77.0	39.9	57.8	63.3	82.0	94.5	78.3	71.34
	+focus+RL	78.1	78.3	40.6	56.7	64.2	82.4	94.2	78.9	71.67

Table 2: All results are presented as the average of three experimental trials. Results marked as * are copied from other papers. Unless otherwise specified, the default experimental setting is 0-shot. HTL(-) represents that the experiment only used Dense Attention and fine-tuning, while "focus" indicates the inclusion of Focus Attention. The "RL" is reinforcement learning.

Model	GSM8K	NumGLUE	MATH	SimulEq	DeepMind	SVAMP	MAWPS	ASDiv	Avg
	СоТ								
MAmmoTH	44.5	36.0	11.86	14.7	34.2	37.0	75.68	60.78	39.34
HTL	44.1	36.2	12.1	15.03	33.8	36.9	75.72	61.0	39.37
Self Distillation									
MAmmoTH(PoT)	58.9	56.6	32.8	44.1	53.7	70.7	91.9	69.3	59.75
HTL(only-PoT)	60.6	59.6	32.7	43.7	52.7	69.7	92.0	71.1	60.26
HTL	61.7	63.0	33.9	48.6	61.1	71.5	92.8	71.6	63.03

Table 3: Based on the performance comparison with Llama-Base, to verify the effectiveness of self-distillation in our experiments.

ing (RL), it is noteworthy that both Llama-Base and Mistral-Base models show improvements in math tasks. Math is currently the most challenging mathematical dataset, generating solutions that are often longer than those of other datasets. This frequently causes the model to repeat generation until it exceeds the limit. Reinforcement learning effectively mitigates this issue. But using RL alone significantly improves performance on similar in-domain datasets, but it struggles to transfer to out-of-domain datasets, and its CTE issue remains unresolved.

Data Ablation We use a self-distillation method to generate data, a technique that has been proven to enhance performance (Zhang et al., 2019). To demonstrate the effectiveness of our approach, we validate the performance of the HTL model in terms of CoT, and we fine-tune the model using

only the PoT subset from the HTL dataset. The results are shown in Table 3. HTL and MAmmoTH exhibit nearly identical performance in CoT, which is in line with our expectations. Our enhancements predominantly arise from the transition from CoT to PoT, rather than from strengthening the capabilities of CoT. And the data from self-distillation only provide a marginal improvement.

3.6 Influence of Subsets

By utilizing training subsets with varying sources and sizes, we can more precisely assess the impact of each data segment on the model's performance. The results show in Table 4. We discover an interesting phenomenon: when we use a specific dataset for downstream training, the model performs well on its corresponding test set but weakens its capabilities on other datasets.

	GSM8K	NumGLUE	MATH	SimulEq	SVAMP	Avg
G	63.3	55.7	32.9	47.4	71.6	54.17
Ν	59.2	64.6	32.4	46.6	70.7	54.7
G/2+N	61.2	62.3	32.8	47.1	69.4	54.76
G/2+N+M/2	62.0	63.9	33.3	48.9	71.9	55.5
G+N+M	61.7	63.0	33.9	48.6	71.5	55.73
2G+N+2M	63.4	62.7	33.7	49.2	71.8	56.16

Table 4: G: GSM8K, M: MATH, N: NumGLUE. G/2 indicates that we only utilized half of the generated GSM8K data, with the aim of exploring whether optimal results can be achieved with a lower data volume. 2G indicates twice the data volume of GSM8K.

	GSM8K	NumGLUE	MATH	SimulEq	SVAMP
Without Inital Tokens	13.3	7.8	1.1	3.2	17.8
Mask Coverage=1	62.7	72.8	33.6	49.6	71.9
Adaptive Training Strategy	63.9	74.1	34.1	50.9	72.3

Table 5: Influence of different Coverage Function.

When we mix multiple datasets for training, the model's improvement in capabilities becomes more comprehensive. The combination of different datasets allows the model to focus more on the characteristics of mathematical problems rather than relying on specific patterns present in only one dataset. At the same time, the addition of GSM8K and NumGLUE has little impact on the MATH dataset; simple mathematical problems are difficult to influence the ability to perform hard reasoning.

Data Volume and Performance Relationship

To explore the appropriate amount of data, we introduced a dataset twice as large for experimentation. The total size of this dataset is 75k, which includes 36,000 entries from GSM8K, 36,000 entries from Math, and 3,000 entries from NumGLUE. As more data is added, the improvement in the model is very slight because we are not injecting more knowledge but rather letting it tend to learn a paradigm.

3.7 The Effort of Mask Coverage Function

The phrase "Without Initial Tokens" indicates that we block all tokens from Q, not preserving the first four, which significantly decreases model performance, almost rendering it unable to reason correctly. In the second experiment, we set the mask coverage to always be 1, not adapting the model during the initial training phase, nor reverting the attention mechanism to a causal mask during the output phase. In this experiment, we find that its loss convergence rate is significantly slower than the Adaptive Training Strategy. The adaptive training strategy performs better across all datasets, serving

Model	Method	Math23k				
CodeLlama						
MAmmoTH	СоТ	14.1				
MAmmoTH	РоТ	34.7				
HTL	-	33.6				
	+focus	30.6				
	+focus+RL	31.8				
	Mistral					
MAmmoTH	СоТ	28.4				
MAmmoTH	РоТ	36.4				
HTL	-	37.9				
	+focus	40.2				
	+focus+RL	40.4				

Table 6: HTL in math23k.

as a transitional phase to balance the Focus Attention training mechanism and the inconsistency during inference. We provide the model with a buffer, allowing it to gradually learn local attention, and after training, we restore it to its autoregressive generation mode. The quadratic function accelerates its speed gradually both during ascent and descent and at the peak, the derivative decreases, resulting in a longer duration of focused attention training.

3.8 Influence of Other Language

We use a Chinese math dataset, math23k (Zhao et al., 2020), to test the advantages of HTL in other languages. The results shows in Table 6 .We observed that the base model's capability in Chinese CoT constrains the effectiveness of HTL, as HTL

relies on effective CoT to enhance PoT. Specifically, when the base model exhibits weak performance in Chinese CoT (e.g., CodeLlama CoT achieves only 14.1% accuracy), the HTL-family methods perform worse than PoT, potentially because the generated Chinese CoT is ineffective so that it rarely helps but rather undermines PoT. Conversely, when the base model has reasonable Chinese CoT capability (e.g., Mistral CoT achieves 28.4% accuracy), the HTL-family methods show better performance than PoT.

4 Analysis

4.1 Error Analysis

To explore how HTL affects model performance and analyze the reasons for errors in various categories, we have divided the errors into two categories: code execution errors and code reasoning errors. Figure 4 shows the proportions of two types of errors across different datasets. For simpler datasets like GSM8K and SVAMP, there are rarely any code execution errors; most are logic reasoning errors, which HTL reduces. For the more challenging dataset like MATH, HTL not only demonstrates stronger logical capabilities but also reduces code execution errors. In HTL, the CTE for CodeLlama-Base and Mistral-Base has been significantly reduced, with CodeLlama-Base decreasing from 8.33% to 3.96% and Mistral-Base from 6.97% to 3.55%. However, the CTE has not been fully resolved because our data only correlates CoT and PoT based on correctness, not process correspondence. In addition to reducing CTE, part of the performance improvement in HTL comes from correctly solving problems that both CoT and PoT got wrong originally.

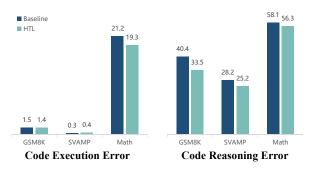


Figure 4: Types of errors and their proportions.

4.2 The Role of Reinforcement Learning

In experiments, the model enhanced with reinforcement learning shows minimal improvement in av-

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erage accuracy (only a 0.3% increase). However, for the MATH dataset, reinforcement learning consistently yields improvements. This improvement stems from reinforcement learning's ability to address the issue of repetitive generation during the CoT in LLMs. When using natural language reasoning, LLMs tend to enumerate answers, leading to repetitive loops until reaching the maximum length limitation. Supervised fine-tuning struggles to suppress this phenomenon, whereas reinforcement learning can effectively penalize it when it occurs.

5 Discussion

Do Larger Models Have Issues with PoT? (Gao et al., 2023) achieved good results in testing on LaMDA-137B and PaLM-540B (Rohan Anil, 2023) by using text to guide code. (Wang et al., 2023a) also employed the method of combining natural language with code, which proved effective on a 70 billion parameter open-source model as well. We conduct evaluations on MAmmoTH-Coder-13B and MAmmoTH-Coder-34B, calculating the proportion of CTE. On the five datasets, MAmmoTH-Coder-13B has an average error rate of 8.2%, while MAmmoTH-Coder-34B has an error rate of 8.7%. CTE does not decrease with the increase in model size. In the future, the amount of training text data will still far exceed that of code data lakes, making it difficult to solve CTE by merely increasing the model size and data volume.

The PoTential of Focused Attention in Other Tasks The current autoregressive inference has limitations in that it cannot obtain the solution to a problem before generating the first token (Gloeckle et al., 2024). CoT can implicitly increase the model's depth, allowing it more extended thinking time to arrive at accurate answers (Feng et al., 2023). Extending the model's thinking time to get the right answer before generating the first valid token will be crucial (Goyal et al., 2023). For reasoning tasks, Focus Attention can gather information and allow large models to concentrate on some intermediate processes (such as setting special tokens) to extend thinking time. On the other hand, Focus Attention can concentrate on the reasoning part of all reasoning tasks while ignoring the question (Q), making the reasoning process more reliable. In several logical/symbolic reasoning tasks, CoT does not significantly outperform directly generating answers (Bao et al., 2024). Focus Attention

may play a crucial role in these cases.

6 Related Work

Current methods primarily rely on the CoT to address mathematical problems, generating reasoning answers in a step-by-step manner (Nye et al., 2021; Imani et al., 2023; Miao et al., 2023; Penedo et al., 2023). The focus of recent research centers on data engineering and prompt design. In the realm of data engineering, the efforts aim to enhance the quality and increase the volume (Luo et al., 2023) of CoT data. However, another stream of research identifies several computational challenges associated with exclusively using the CoT approach. In response, (Chen et al., 2023) introduces the PoT, a method that employs Python programs to achieve more accurate results. (Yue et al., 2023) attempts to merge CoT and PoT data in their dataset, resulting in significant improvements. Similarly, (Gao et al., 2023) seeks to enhance PoT with CoT by weaving CoT snippets into code lines, employing few-shot prompting to guide the model. ToRA (Gou et al., 2023b) uses imitation learning to allow natural language to correct errors in code. MathCoder (Wang et al., 2023a) improves accuracy by closely integrating natural language with code, distilling large amounts of data from GPT-4. OpenmathInstruct (Toshniwal et al., 2024) employs Mistral to explore various problem-solving approaches for each GSM8K and MATH problem, providing a 1.8M open-source dataset to the community.

7 Conclusion

In our paper, we identify CTE in mathematical problems and explore how to address the gap between large models' text and code capabilities through text and code interaction. We propose HTL, a method that can more closely integrate CoT and PoT to achieve more accurate reasoning and avoid calculation errors. Our experiment shows that without introducing additional information, our method achieves excellent results merely by controlling the flow of information.

Limitations

Lack of Equipment Due to GPU limitations, our experiments are only conducted on the 7B model, and we did not attempt larger models like the 34B or 70B. Although we provide theoretical feasibility, there is a lack of practical experimental support.

Data Relevance The CoT and PoT data constructed through automated methods are only associated based on correctness. We still lack human evaluation to determine whether their reasoning processes correspond accurately. This is evident from our experimental results: there is a significant improvement for simpler datasets like GSM8K, as the problem-solving approaches are generally similar. However, for more challenging datasets that may have multiple different solutions, the relevance might be lower.

Exploration of Focus Attention Regarding Focus Attention, we have not yet determined the specific reason for the need for a gradual increase in coverage to adapt to inference. If we extend this approach to other domains, such as incorporating it into the pre-training stage, it enables the model to better learn step-by-step generation, PoTentially leading to improved results.

Experimental Limitations with Closed-source Models We conduct experiments only on opensource large models. Due to the cost of running the models in the API, we do not attempt to explore whether this paradigm can enhance the inference capabilities of models like GPT-4 by constructing similar prompts. However, in our experiments, merely adjusting the prompts does not result in a significant performance improvement. Furthermore, because the training data for GPT-4 is unknown, its results are challenging to interpret.

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