Unified Interpretation of Smoothing Methods for Negative Sampling Loss Functions in Knowledge Graph Embedding

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Abstract

Knowledge Graphs (KGs) are fundamental resources in knowledge-intensive tasks in NLP. Due to the limitation of manually creating KGs, KG Completion (KGC) has an important role in automatically completing KGs by scoring their links with KG Embedding (KGE). To handle many entities in training, KGE relies on Negative Sampling (NS) loss that can reduce the computational cost by sampling. Since the appearance frequencies for each link are at most one in KGs, sparsity is an essential and inevitable problem. The NS loss is no exception. As a solution, the NS loss in KGE relies on smoothing methods like Self-Adversarial Negative Sampling (SANS) and subsampling. However, it is uncertain what kind of smoothing method is suitable for this purpose due to the lack of theoretical understanding. This paper provides theoretical interpretations of the smoothing methods for the NS loss in KGE and induces a new NS loss, Triplet Adaptive Negative Sampling (TANS), that can cover the characteristics of the conventional smoothing methods. Experimental results of TransE, DistMult, ComplEx, RotatE, HAKE, and HousE on FB15k-237, WN18RR, and YAGO3-10 datasets and their sparser subsets show the soundness of our interpretation and performance improvement by our TANS.

1 Introduction

Knowledge Graphs (KGs) represent human knowledge using various entities and their relationships as graph structures. KGs are fundamental resources for knowledge-intensive tasks like dialog (Moon et al., 2019), question answering (Reese et al., 2020), named entity recognition (Liu et al., 2019), open-domain questions (Hu et al., 2022), and recommendation systems (Gao et al., 2020), etc.

However, to create complete KGs, we need to consider a large number of entities and all their possible relationships. Taking into account the explosively large number of combinations between entities, only relying on manual approaches is unrealistic to make complete KGs.

Knowledge Graph Completion (KGC) is a task to deal with this problem. KGC involves automatically completing missing links corresponding to relationships between entities in KGs. To complete the KGs, we need to score each link between entities. For this purpose, current KGC commonly relies on Knowledge Graph Embedding (KGE) (Bordes et al., 2011). KGE models predict the missing relations, named link prediction, by learning structural representations. In the current KGE, models need to complete a link (triplet) (e_i, r_k, e_i) of entities e_i and e_j , and their relationship r_k by answering e_i or e_j from a given query $(?, r_k, e_j)$ or $(e_i, r_k, ?)$, respectively. Hence, KGE needs to handle a large number of entities and their relationships during its training.

To handle a large number of entities and relationships in KGs, Negative Sampling (NS) loss (Mikolov et al., 2013) is frequently used for training KGE models. The original NS loss is proposed to approximate softmax cross-entropy loss to reduce computational costs by sampling false labels from its noise distribution in training. Trouillon et al. (2016) import the NS loss from word embedding to KGE with utilizing uniform distribution as its noise distribution. Sun et al. (2019) extend the NS loss to Self-Adversarial Negative Sampling (SANS) loss for efficient training of KGE. Unlike the NS loss with uniform distribution, the SANS loss utilizes the training model's prediction as the noise distribution. Since the negative samples in the SANS loss become more difficult to discriminate for models in training, the SANS can extract models' potential compared with the NS loss with uniform distribution.

One of the problems left for KGE is the sparsity of KGs. Figure 1 shows the appearance frequency of queries and answers (entities) in the training data of FB15k-237, WN18RR and YAGO3-10 datasets.

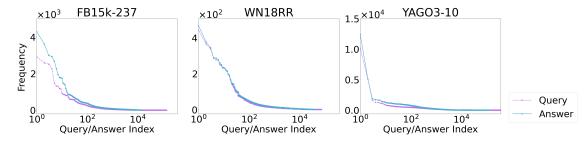


Figure 1: Appearance frequencies of queries and answers (entities) in the training data of FB15k-237, WN18RR, and YAGO3-10. Note that the indices are sorted from high frequency to low.

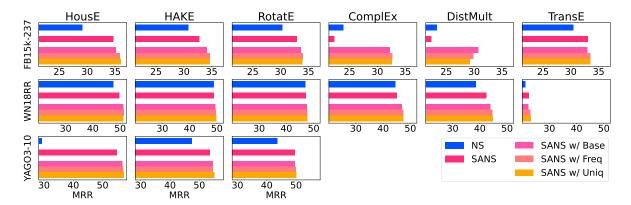


Figure 2: Performances of KGE models HousE, HAKE, RotatE, ComplEx, DistMult, and TransE on datasets FB15k-237, WN18RR, and YAGO3-10 using NS, SANS, and subsampling methods (noted as *Base, Freq, Uniq*).

From the long-tail distribution of this figure, we can understand that both queries and answers necessary for training KGE models may suffer from the sparsity problem.

As a solution, several smoothing methods are used in KGE. Sun et al. (2019) import subsampling from word2vec (Mikolov et al., 2013) to KGE. Subsampling can smooth the appearance frequency of triplets and queries in KGs. Kamigaito and Hayashi (2022a) show a general formulation that covers the basic subsampling of Sun et al. (2019) (Base), their frequency-based subsampling (Freq) and uniquebased subsampling (Uniq) for KGE. Kamigaito and Hayashi (2021) indicate that SANS has a similar effect of using label-smoothing (Szegedy et al., 2016) and thus SANS can smooth the frequencies of answers in training. Figure 2 shows the effectiveness of SANS and subsampling in KGC performance. From the figure, since FB15k-237 is more sparse (imbalanced) than WN18RR and YAGO3-10 based on Figure 1, we can understand that strategy in choosing smoothing methods have more considerable influences than models when data is sparse.

While SANS and subsampling can improve model performance by smoothing the appearance frequencies of triplets, queries, and answers, their theoretical relationship is not clear, leaving their capabilities and deficiencies a question. For example, conventional works (Sun et al., 2019; Zhang et al., 2020b; Kamigaito and Hayashi, 2022a)¹ jointly use SANS and subsampling with no theoretical background. Thus, there is a call for further interpretability and performance improvement.

To solve the above problem, we theoretically and empirically study the differences of SANS and subsampling on three common datasets and their sparser subsets with six popular KGE models². Our contributions are as follows:

- By focusing on the smoothing targets, we theoretically reveal the differences between SANS and subsampling and induce a new NS loss, Triplet Adaptive Negative Sampling (TANS), that can cover the smoothing target of both SANS and subsampling.
- We theoretically show that TANS with subsampling can potentially cover the conven-

¹Note that Sun et al. (2019); Zhang et al. (2020b) use subsampling in their released implementation without referring to it in their paper.

²Our code and data are available at https://github.com/xincanfeng/ss_kge.

tional usages of SANS and subsampling.

- We empirically verify that TANS improves KGC performance on sparse KGs in terms of MRR.
- We empirically verify that TANS with subsampling can cover the conventional usages of SANS and subsampling in terms of MRR.

2 Background

In this section, we describe the problem formulation for solving KGC by KGE and explain the conventional NS loss functions in KGE.

2.1 Formulation of KGE

KGC is a research topic for automatically inferring new links in a KG that are likely but not yet known to be true. To infer the new links by KGE, we decompose KGs into a set of triplets (links). By using entities e_i , e_j and their relation r_k , we represent the triplet as (e_i, r_k, e_j) . In a typical KGC task, a KGE model receives a query $(e_i, r_k, ?)$ or $(?, r_k, e_j)$ and predicts the entity corresponding to ? as an answer.

In KGE, a KGE model scores a triplet (e_i, r_k, e_j) by using a scoring function $s_{\theta}(x, y)$, where θ denotes model parameters. Here, using a softmax function, we represent the existence probability $p_{\theta}(y|x)$ for an answer y of the query x as follows:

$$p_{\theta}(y|x) = \frac{\exp(s_{\theta}(x,y))}{\sum_{y' \in Y} \exp(s_{\theta}(x,y'))}, \quad (1)$$

where Y is a set of entities.

2.2 NS Loss in KGE

To train $s_{\theta}(x,y)$, we need to calculate losses for the observables $D = \{(x_1,y_1),\cdots,(x_n,y_n)\}$ that follow $p_d(x,y)$. Even if we can represent KGC by Eq. (1), it does not mean we can tractably perform KGC due to the large number of Y in KGs. For the reason of the computational cost, the NS loss (Mikolov et al., 2013) is used to approximate Eq. (1) by sampling false answers.

By modifying that of Mikolov et al. (2013), the following NS loss (Sun et al., 2019; Ahrabian et al., 2020) is commonly used in KGE:

$$\ell_{NS}(\theta) = -\frac{1}{|D|} \sum_{(x,y)\in D} \left[\log(\sigma(s_{\theta}(x,y) + \tau)) + \frac{1}{\nu} \sum_{y_{i}\sim U}^{\nu} \log(\sigma(-s_{\theta}(x,y_{i}) - \tau)) \right], \quad (2)$$

where U is the noise distribution that follows uniform distribution, σ is the sigmoid function, ν is the number of negative samples per positive sample (x, y), and τ is a margin term to adjust the value range decided by $s_{\theta}(x, y)$.

2.3 Smoothing Methods for the NS Loss in KGE

As shown in Figure 1, KGC needs to deal with the sparsity problem caused by low frequent queries and answers in KGs. Imposing smoothing on the appearance frequencies of queries and answers can mitigate this problem. The following subsections introduce subsampling (Mikolov et al., 2013; Sun et al., 2019; Kamigaito and Hayashi, 2022a) and SANS (Sun et al., 2019), the conventional smoothing methods for the NS loss in KGE.

2.3.1 Subsampling

Subsampling (Mikolov et al., 2013) is a method to smooth the frequency of triplets or queries in the NS loss. Sun et al. (2019) import this approach from word embedding to KGE. Kamigaito and Hayashi (2022b,a) add some variants to subsampling for KGC and theoretically provide a unified expression of them as follows:

$$\ell_{\text{SUB}}(\theta)$$

$$= -\frac{1}{|D|} \sum_{(x,y)\in D} \left[A(x,y;\alpha) \log(\sigma(s_{\theta}(x,y) + \tau)) + \frac{1}{\nu} \sum_{y_{i}\sim U}^{\nu} B(x,y;\alpha) \log(\sigma(-s_{\theta}(x,y_{i}) - \tau)) \right], (3)$$

where α is a temperature term to adjust the frequecy of triplets and queries. Note that we incorporate α into Eq. (3) to consider various loss functions even though Kamigaito and Hayashi (2022b,a) do not consider α . In this formulation, we can consider several assumptions for deciding $A(x,y;\alpha)$ and $B(x,y;\alpha)$. We introduce these assumptions in the following paragraphs:

Base As a basic subsampling approach, Sun et al. (2019) import the one originally used in word2vec (Mikolov et al., 2013) to KGE, defined as follows:

$$A(x,y;\alpha) = B(x,y;\alpha) = \frac{\#(x,y)^{-\alpha}|D|}{\sum_{(x,y')\in D} \#(x',y')^{-\alpha}},$$
(4)

where # is the symbol for frequency and #(x,y) represents the frequency of (x,y). In word2vec,

subsampling randomly discards a word by a probability $1-\sqrt{t/f}$, where t is a constant value and f is a frequency of a word. This is similar to randomly keeping a word with a probability $\sqrt{t/f}$. Thus, we can understand that Eq. (4) follows the original use in word2vec. Since the actual (x, y) occurs at most once in KGs, when $(x, y) = (e_i, r_k, e_j)$, they approximate the frequency of (x, y) as:

$$\#(x,y) \approx \#(e_i, r_k) + \#(r_k, e_j),$$
 (5)

based on the approximation of n-gram language modeling (Katz, 1987).

Freq Kamigaito and Hayashi (2022a) propose frequency-based subsamping (Freq) by assuming a case that (x, y) originally has a frequency, but the observed one in the KG is at most 1.

$$A(x, y; \alpha) = \frac{\#(x, y)^{-\alpha} |D|}{\sum_{(x', y') \in D} \#(x', y')^{-\alpha}},$$

$$B(x, y; \alpha) = \frac{\#x^{-\alpha} |D|}{\sum_{x' \in D} \#x'^{-\alpha}}.$$
(6)

Uniq Kamigaito and Hayashi (2022a) also propose unique-based subsamping (Uniq) by assuming a case that the originally frequency and the observed one in the KG are both 1.

$$A(x, y; \alpha) = B(x, y; \alpha) = \frac{\#x^{-\alpha}|D|}{\sum_{x' \in D} \#x'^{-\alpha}}.$$
 (7)

2.3.2 SANS Loss

SANS is originally proposed as a kind of NS loss to train KGE models efficiently by considering negative samples close to their corresponding positive ones. Kamigaito and Hayashi (2021) show that using SANS is similar to imposing label-smoothing on Eq. (1). Thus, SANS is a method to smooth the frequency of answers in the NS loss. The SANS loss is represented as follows:

$$\begin{split} &\ell_{\text{SANS}}(\theta) \\ &= -\frac{1}{|D|} \sum_{(x,y) \in D} \Big[\log(\sigma(s_{\theta}(x,y) + \tau)) \\ &+ \sum_{y_{i} \sim U}^{\nu} p_{\theta}(y_{i}|x;\beta) \log(\sigma(-s_{\theta}(x,y_{i}) - \tau)) \Big], \ \ (8) \end{split}$$

$$p_{\theta}(y_i|x;\beta) \approx \frac{\exp(\beta s_{\theta}(x,y_i))}{\sum_{j=1}^{\nu} \exp(\beta s_{\theta}(x,y_j))},$$
 (9)

where β is a temperature to adjust the distribution of negative sampling. Different from subsampling, SANS uses $p_{\theta}(y_i|x;\beta)$ that is predicted by

a model θ to adjust the frequency of the answer y_i . Since $p_{\theta}(y_i|x;\beta)$ is essentially a noise distribution, it does not receive any gradient during training.

3 Triplet Adaptive Negative Sampling

In this section, we explain our proposed Triplet Adaptive Negative Sampling (TANS) in detail. We first show the overview of our TANS through the comparison with the conventional smoothing methods of the NS loss for KGE (See §2.3) in §3.1 and after that we explain the details of TANS through its mathematical formulations in §3.2 and §3.3.

3.1 Overview

TANS is fundamentally different from SANS, with SANS only taking into account the conditional probability of negative samples and TANS being a loss function that considers the joint probability of the pair of queries and their answers.

Table 1 shows the characteristics of TANS and the conventional smoothing methods of the NS loss for KGE introduced in §2.3. These characteristics are based on the decomposition of $p_d(x, y)$, the appearance probability for the triplet (x, y), into that of its answer $p_d(y|x)$ and query p(x):

$$p_d(x,y) = p_d(y|x)p_d(x) \tag{10}$$

In Eq. (10), smoothing both $p_d(y|x)$ and $p_d(x)$ is similar to smoothing $p_d(x,y)$. However, smoothing $p_d(x,y)$ does not ensure smoothing both $p_d(x)$ and $p_d(y|x)$ considering the case of only one of them being smoothed, and the left one being still sparse. Similarly, smoothing only $p_d(x)$ or $p_d(y|x)$ does not ensure $p_d(x,y)$ being smoothed due to the case where one of them is still sparse. In Table 1, we denote such a case where the method can influence the probability, but no guarantee of the probability be smoothed as \triangle .

In TANS, we aim to smooth $p_d(x, y)$ by smoothing both $p_d(y|x)$ and $p_d(x)$ based on Eq. (10).

3.2 Formulation

Here, we induce TANS from SANS with targeting to smooth $p_d(x, y)$ by smoothing both $p_d(y|x)$ and $p_d(x)$. First, we assume a simple replacement from $p_\theta(y|x)$ to $p_\theta(x, y)$ in $\ell_{\text{SANS}}(\theta)$ of Eq. (9):

$$-\frac{1}{|D|} \sum_{(x,y)\in D} \left[\log(\sigma(s_{\theta}(x,y) + \tau)) + \sum_{x_i \in U}^{\nu} p_{\theta}(x,y_i) \log(\sigma(-s_{\theta}(x,y_i) - \tau)) \right]. \tag{11}$$

Mathad	Method		moothing		- Remarks
Method		p(x,y)	p(y x)	p(x)	Kemarks
Subsampling	Base Uniq Freq	✓ △ ✓	△ × △	△ ✓ ✓	p(y x) and $p(x)$ are influenced by $p(x,y)$. $p(x,y)$ is indirectly controlled by $p(x)$. $p(y x)$ is indirectly controlled by $p(x,y)$ or $p(x)$.
SANS		Δ	✓	×	p(x,y) is indirectly controlled by $p(y x)$.
TANS		✓	✓	✓	

Table 1: The characteristics of each smoothing method for the NS loss in KGE (See §2.3 for the details.) and our proposed TANS. \checkmark and \triangle respectively denote the method smooths the probability directly and indirectly. \times denotes the method does not smooth the probability.

However, using Eq. (11) causes an imbalanced loss between the first and second terms since the sum of $p_{\theta}(x, y_i)$ on all negative samples is not always 1. Thus, Eq. (11) is impractical as a loss function.

As a solution, we focus on the decomposition $p_{\theta}(x,y) = p_{\theta}(y|x)p_{\theta}(x)$ and the fact that the sum of $p_{\theta}(y|x)$ of all negative samples is always 1. By using $p_{\theta}(x)$ to make a balance between the first and second loss term, we can modify Eq. (11) and induce our TANS as follows:

$$\ell_{\text{TANS}}(\theta) = -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(x;\gamma) \Big[\log(\sigma(s_{\theta}(x,y) + \tau)) + \sum_{y_{i}\sim U}^{\nu} p_{\theta}(y_{i}|x;\beta) \log(\sigma(-s_{\theta}(x,y_{i})-\tau)) \Big], \quad (12)$$

$$p_{\theta}(x;\gamma) = \sum_{y_{i}\in D} p_{\theta}(x,y_{i};\gamma),$$

$$p_{\theta}(x,y_{i};\gamma) = \frac{\exp(\gamma s_{\theta}(x,y_{i}))}{\sum_{(x',y')\in D} \exp(\gamma s_{\theta}(x',y'))}, \quad (13)$$

where γ is a temperature to smooth the frequency of queries. Since TANS uses a noise distribution decided by $p_{\theta}(x; \gamma)$ and $p_{\theta}(y_i|x; \beta)$, it does not propagate gradients through probabilities for negative samples, and thus, memory usage is not increased.

3.3 Theoretical Interpretation

In this subsection, we discuss the difference and similarities among TANS and other smoothing methods for the NS loss in KGE. As shown in Table 1, the subsampling methods, Base and Freq, can smooth triplet frequencies similar to our TANS. To investigate TANS from the view point of sub-

sampling, we reformulate Eq. (12) as follows:

$$\ell_{\text{TANS}}(\theta)$$

$$= -\frac{1}{|D|} \sum_{(x,y)\in D} \left[A(x,y;\gamma) \log(\sigma(s_{\theta}(x,y)+\tau)) + \sum_{y_{i}\sim U}^{\nu} B(x,y;\beta,\gamma) \log(\sigma(-s_{\theta}(x,y_{i})-\tau)) \right],$$

$$A(x,y;\gamma) = p_{\theta}(x;\gamma),$$

$$B(x,y;\beta,\gamma) = p_{\theta}(y_{i}|x;\beta)p_{\theta}(x;\gamma).$$
(15)

Apart from the temperature terms, α , β , and γ , we can see that the general formulation of subsampling in Eq. (3) and the above Eq. (14) has the same formulation. Thus, TANS is not merely an extension of SANS but also a novel subsampling method.

Even though their similar characteristic, TANS and subsampling have an essential difference: TANS smooths the frequencies by model-predicted distributions as in Eq. (13), and the subsampling methods smooth them by counting appearance frequencies on the observed data as in Eq. (4), (5), (6), and (7). For instance, TANS can work even when the entity or relations included in the target triplet appear more than once, which is theoretically different from conventional approaches.

Since the superiority of using either model-based or count-based frequencies depends on the model and dataset, we empirically investigate this point through our experiments.

4 Unified Interpretation of SANS and Subsampling

In the previous section, we understand that our TANS can smooth triplets, queries, and answers partially covered by SANS and subsampling methods. On the other hand, TANS only relies on model-predicted frequencies to smooth the frequencies.

Те	Temperature		Induced NS Loss
α	β	γ	
=0	=0	=0	Equivalent to $\ell_{NS}(\theta)$, the basic NS loss in KGE (Eq. (2))
=0	=0	$\neq 0$	Currently does not exist
=0	$\neq 0$	=0	Proportional to $\ell_{\text{SANS}}(\theta)$, the SANS loss (Eq. (9))
=0	$\neq 0$	$\neq 0$	Equivalent to our $\ell_{TANS}(\theta)$, the TANS loss (Eq. (12))
$\neq 0$	=0	=0	Proportional to $\ell_{NS}(\theta)$, the basic NS loss in KGE (Eq. (2)) with subsampling in §2.3
$\neq 0$	=0	$\neq 0$	Currently does not exist
$\neq 0$	$\neq 0$	=0	Proportional to $\ell_{\text{SANS}}(\theta)$, the SANS loss (Eq. (9)) with subsampling in §2.3
$\neq 0$	$\neq 0$	$\neq 0$	Equivalent to our $\ell_{\text{UNI}}(\theta)$, the unified NS loss in KGE (Eq. (16))
			and also equivalent to our $\ell_{TANS}(\theta)$, the TANS loss (Eq. (12)) with subsampling in §2.3

Table 2: The relationship among the loss functions from the viewpoint of the unified NS loss, $\ell_{\text{UNI}}(\theta)$ in Eq. (16).

Neubig and Dyer (2016) point out the benefits of combining count-based and model-predicted frequencies in language modeling. This section integrates smoothing methods for the NS loss in KGE from a unified interpretation.

4.1 Formulation

We formulate the unified loss function by introducing subsampling (Eq. (3)) into our TANS (Eq. (12)) as follows:

$$\ell_{\text{UNI}}(\theta) = -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(x;\gamma) \Big[A(x,y;\alpha) \log(\sigma(s_{\theta}(x,y)+\tau)) \Big]$$

$$= -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(x;\gamma) \Big[A(x,y;\alpha) \log(\sigma(s_{\theta}(x,y)+\tau)) \Big]$$

$$= -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(x;\gamma) \Big[A(x,y;\alpha) \log(\sigma(s_{\theta}(x,y)+\tau)) \Big]$$

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$$= -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(y;\alpha) \Big[A(x,y;\alpha) \exp(s_{\theta}(x,y)+\tau) \Big]$$

$$= -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(y;\alpha) \Big[A(x,y) \exp(s_{\theta}(x,y) \Big]$$

$$= -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(x,y) \Big[A(x,y) \exp(s_{\theta}(x,y) \Big]$$

$$= -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(y;\alpha) \Big[A(x,y) \exp(s_{\theta}$$

where η is a hyperparamter that can be any value to absorb the difference among the three different subsampling methods, Base, Uniq, and Freq.

Here, we can induce the NS losses shown in our paper from Eq. (16) by changing the temperature parameters α , β , and γ . Table 2 shows the induced losses from our $\ell_{\text{UNI}}(\theta)$. Note that since $p_{\theta}(x; \gamma)$ only appears in our TANS, canceling $p_{\theta}(x; \gamma)$ by $\gamma = 0$ induces an inequivalent but a proportional relationship to the conventional NS loss.

Theoretical Interpretation

As shown in Table 2, TANS w/ subsampling has characteristics of all smoothing methods for the NS loss in KGE introduced in this paper. Therefore, we can expect higher performance of TANS w/ subsampling than the combination of conventional methods, the basic NS, SANS, and subsampling. However, because TANS w/ subsampling uses subsampling in §2.3, we need to choose the one from

Base, Uniq, and Freq for TANS w/ subsampling. Since this part is out of the scope of theoretical interpretation, we investigate this in the experiments.

Experiments

In this section, we investigate our theoretical interpretation in §3.3 and §4.2 through experiments.

Experimental Settings

we used TransE (Bordes et al., 2013), Dist-Mult (Yang et al., 2015), ComplEx (Trouillon et al., 2016), RotatE (Sun et al., 2019), HAKE (Zhang et al., 2020a), and HousE (Li et al., 2022). We followed the original settings of Sun et al. (2019) for TransE, DistMult, ComplEx, and RotatE with their implementation⁴, the original settings of Zhang et al. (2020a) for HAKE with their implementation⁵, and the original settings of Li et al. (2022) for HousE with their implementation⁶. We tuned temperature γ on the validation split for each dataset.

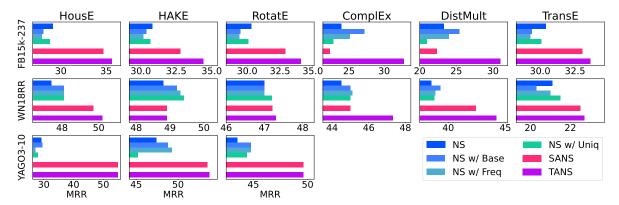
Metrics We employed conventional metrics in KGC, i.e., MRR, Hits@1 (H@1), Hits@3 (H@3), and Hits@10 (H@10) and reported the average scores and their standard deviations by three different runs with fixed random seeds.

³Table 3 in Appendix A shows the dataset statistics.

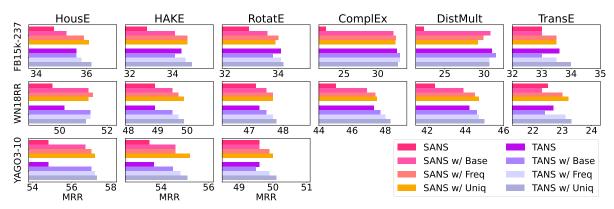
⁴https://github.com/DeepGraphLearning/ KnowledgeGraphEmbedding

⁵https://github.com/MIRALab-USTC/

⁶https://github.com/rui9812/HousE



(a) Results on datasets FB15k-237, WN18RR, YAGO3-10 using NS, SANS, TANS, and NS with subsampling.



(b) Results on datasets FB15k-237, WN18RR, YAGO3-10 using SANS, TANS, and those with subsampling.

Figure 3: KGC performance on common KGs (Notations are the same as in Figure 2).

5.2 Results

Since the result tables are large⁷, we discuss them individually, focusing on important information in the following subsections.

5.2.1 Effectiveness of TANS

Figure 3a shows the MRR scores of each method. From the result, we can understand the effectiveness of considering triplet information in SANS as conducted in TANS. Thus, the result is along with our expectation in §3.3 that TANS can cover the role of subsampling methods. However, as the result of HAKE on WN18RR shows, there is a case that subsampling methods outperform TANS. As discussed in §3.3, using only TANS does not cover all combinations of NS loss and subsampling. Considering this theoretical fact, we further compare TANS with subsampling and the SANS loss with subsampling in the following section.

5.2.2 Validity of the Unified Interpretation

Figure 3b shows the result for each configuration. We can see performance improvements by using subsampling in both SANS and TANS. Furthermore, in almost all cases, TANS with subsampling achieve the highest MRR. This observation is along with the theoretical conclusion in §3.3 that TANS with subsampling can cover the characteristic of other NS loss in terms of smoothing. On the other hand, the results of HAKE on YAGO3-10 show the different tendency that SANS with subsampling achieves the best MRR instead of TANS. Because the model prediction estimates the triplet frequencies, TANS is influenced by the selected model. Therefore, carefully choosing the combination of a loss function and model is still effective in improving KGC performance on the NS loss with subsampling.

6 Analysis

We analyze how TANS mitigates the sparsity problem in imbalanced KGs commonly caused by low frequent triplets in KGC. By considering that all

⁷The full experimental results are listed in Appendix B. The scores are included in Table 5, 6, and 7 of Appendix B.1. The training loss curves and validation MRR curves for each smoothing method are in Figure 6, 7, and 8 of Appendix B.2.

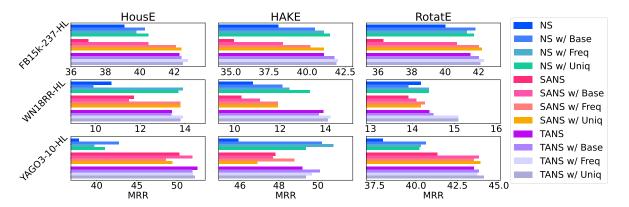


Figure 4: KGC performance on filtered sparser KGs, i.e., FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL (Notations are the same as in Figure 2).

triplets in KGs appear at most once, we focus on queries. We extracted 0.5% triplets with the highest or lowest frequent queries in training, validation, and test splits as the sparser subsets FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL, respectively ⁸ from original data, for the investigation.

Figure 4 shows MRRs for each model on each sparser dataset. From the result, we can understand that TANS can perform even much better in KGC when KGs get more imbalanced. You can see further detailed results in Table 8, 9, and 10 of Appendix C.3.

7 Related Work

Knowledge Graph Knowledge graphs have important roles in various knowledge-intensive NLP tasks like dialog (Moon et al., 2019), question answering (Reese et al., 2020), named entity recognition (Liu et al., 2019), open-domain questions (Hu et al., 2022), recommendation systems (Gao et al., 2020), and commonsense reasoning (Sakai et al., 2024b), etc. In addition to these text-only tasks, knowledge-intensive vision and language (V&L) tasks such as visual question answering (VQA) (Yue et al., 2023), image generation (Kamigaito et al., 2023), explanation generation (Hayashi et al., 2024), and image review generation (Saito et al., 2024) also require external knowledge. Visual KGs (Zhu et al., 2024) have the potential to contribute to solving these tasks. Therefore, KGs are important materials in various different fields.

Knowlege Graph Completion Even though KGs are useful, their sparsity is a fundamental prob-

lem. To solve the sparsity of knowledge graphs, we need to complete them by inferring their unseen links between nodes, which are entities. For that purpose, knowledge graph completion (KGC) and knowledge graph embedding (KGE) (Bordes et al., 2011), which represents KG information as a continuous vector space, are commonly used. As KGE methods, vector space models like TransE (Bordes et al., 2013), DistMult (Yang et al., 2015), ComplEx (Trouillon et al., 2016), RotatE (Sun et al., 2019), HAKE (Zhang et al., 2020a), and HousE (Li et al., 2022), that learn only from task-specific datasets expand this field as pioneers. As well as such approaches, pre-trained language model (PLM)-based approaches like KEPLER (Wang et al., 2021) and SimKGC (Wang et al., 2022) also have an important role in KGC due to their ability to utilize the knowledge obtained in pretraining. However, as pointed out by Sakai et al. (2024a), PLM-based approaches have a leakage issue caused by data contamination in pre-training. Generation-based KGC methods like KGT5 (Saxena et al., 2022) and GenKGC (Xie et al., 2022) are unique in directly generating entity names. In hierarchical text classification (HTC), generation-based approaches contribute to improving performance (Kwon et al., 2023) supported by considering label hierarchies by fusing pre-trained text and label embeddings (Xiong et al., 2021; Zhang et al., 2021) on the decoder. However, Sakai et al. (2024a) point out that commonly used KGC methods conduct link-level prediction, and such generation-based KGC methods make it difficult to use structure information of KGs directly. Thus, their performance gain is limited. This situation requires investigating the benefits of inferring links by generation-based

⁸Note that we show their appearance frequencies of queries and answers in the training data in Figure 5 and detailed statistics in Table 4 of Appendix C.1 and C.2, respectively.

KGC under predefined entities and relationships.

Negative Sampling Mikolov et al. (2013) initially propose the NS loss of the frequent words to train their word embedding model, word2vec. Trouillon et al. (2016) introduce the NS loss to KGE to speed up training. Melamud et al. (2017) use the NS loss to train the language model. In contextualized pre-trained embeddings, Clark et al. (2020a) indicate that a BERT (Devlin et al., 2019)like model ELECTRA (Clark et al., 2020b) uses the NS loss to perform better and faster than language models. Sun et al. (2019) extend the NS loss to SANS loss for KGE and propose their noise distribution, which is subsampled by a uniformed probability $p_{\theta}(y_i|x)$. Kamigaito and Hayashi (2021) point out the sparseness problem of KGs through their theoretical analysis of the NS loss in KGE. Furthermore, Kamigaito and Hayashi (2022a,b) reveal that subsampling (Mikolov et al., 2013) can alleviate the sparseness problem in the NS for KGE and conclude three assumptions for subsampling, i.e., Base, Freq, and Uniq. Feng et al. (2023) incorporate their proposed model-based subsampling that estimates frequencies for entities and their relationships by a trained KGE model into the subsampling of the NS loss to mitigate the sparseness issue of counting the frequency by increasing computational cost to train the additional KGE model.

Our Work Through our work, we theoretically clarify the position of the previous works on SANS loss and subsampling from the viewpoint of smoothing methods for the NS loss in KGE. Since our work unitedly interprets SANS loss and subsampling, our proposed TANS inherits the advantages of conventional works and can deal with the sparsity problem in the NS loss for KGE.

8 Conclusion

We reveal the relationships between SANS loss and subsampling for the KG completion task through theoretical analysis. We explain that SANS loss and subsampling under three assumptions, Base, Freq, and Uniq have similar roles to mitigate the sparseness problem of queries and answers of KGs by smoothing the frequencies of queries and answers. Furthermore, based on our interpretation, we induce a new loss function, Triplet Adaptive Negative Sampling (TANS), by integrating SANS loss and subsampling. We also introduce a theoretical interpretation that TANS with subsampling can

cover all conventional combinations of SANS loss and subsampling.

We verified our interpretation by empirical experiments in three common datasets, FB15k-237, WN18RR, and YAGO3-10, and six popular KGE models, TransE, DistMult, ComplEx, RotatE, HAKE, and HousE. The experimental results show that our TANS loss can outperform subsampling and SANS loss with many models in terms of MRR as expected by our theoretical interpretation. Furthermore, the combinatorial use of TANS and subsampling achieved comparable or better performance than other combinations and showed the validity of our theoretical interpretation that TANS with subsampling can cover all conventional combinations of SANS loss and subsampling in KGE.

Limitations

Our experiments are conducted exclusively on public datasets, which are relatively well-balanced. Consequently, we anticipate that our TANS will perform better on real-world KGs.

Ethics Statement

We used the publicly available datasets, FB15k-237, WN18RR, and YAGO3-10, to train and evaluate KGE models, and there is no ethical consideration.

Reproducibility Statement

We used the publicly available code to implement KGE models, TransE, DistMult, ComplEx, RotatE, HAKE, and HousE with the author-provided hyperparameters as described in $\S 5.1$. Regarding the temperature parameter γ , we tuned it on the validation split for each dataset and reported the values in Table 5, 6, and 7 of Appendix B. Our code and data are available at https://github.com/xincanfeng/ss_kge.

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A Dataset Statistics

Table 3 shows the dataset statistics for dataset FB15k-237, WN18RR, and YAGO3-10, introduced in §5.1.

B Full Experimental Results

B.1 Results Tables

Table 5, 6, and 7 list all results on FB15k-237, WN18RR, and YAGO3-10, explained in §5.2. In these tables, the bold scores are the best results for each subsampling type (e.g. *None*, *Base*, *Freq*, and *Uniq*.), \dagger indicates the best scores for each model, *SD* denotes the standard deviation of the three trials, and γ denotes the temperature chosen by development data.

B.2 Training Loss and Validation MRR Curve

Figure 6, 7, and 8 show the training loss curves and validation MRR curves for each smoothing method. From these figures, we can understand that the convergence of TANS loss is as well as SANS and NS loss on datasets FB15k-237, WN18RR, and YAGO3-10 for each KGE model. Meanwhile, the time complexity of TANS is the same with SANS and NS loss too.

C Sparse Queries

C.1 Appearance Frequencies of Queries and Answers

Figure 5 shows the appearance frequencies of queries and answers in the training set of our filtered sparser data FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL, expained in §6.

C.2 Data Statistics

Table 4 shows detailed statistics of our filtered sparser data FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL, expained in §6.

C.3 Detailed Results

Table 8, 9, and 10 shows the detailed results on our filtered sparser data FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL, expained in §6. Notations are as those described in §B.1.

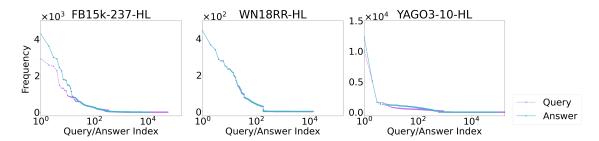


Figure 5: Appearance frequencies of queries and answers (entities) in the training data of the sparser subsets FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL. Note that the indices are sorted from high frequency to low.

Dataset	Split	Tuple	Query	Entity	Relation
	Total	310,116	150,508	14,541	237
FB15k-237	#Train	272,115	138,694	14,505	237
FB13K-237	#Valid	17,535	19,750	9,809	223
	#Test	20,466	22,379	10,348	224
	Total	93,003	77,479	40,943	11
WN18RR	#Train	86,835	74,587	40,559	11
WNISKK	#Valid	3,034	5,431	5,173	11
	#Test	3,134	5,565	5,323	11
	Total	1,089,040	372,775	123,182	37
YAGO3-10	#Train	1,079,040	371,077	123,143	37
1AGO3-10	#Valid	5,000	8,534	7,948	33
	#Test	5,000	8,531	7,937	34

Tabl	e 3:	Statistics	tor	each	pub	lic (dataset.
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Dataset	Split	Tuple	Query	Entity	Relation
	Total	111,631	63,330	11,828	155
ED 151 227 111	#Train	95,244	55,923	11,600	155
FB15k-237-HL	#Valid	7,571	6,918	4,933	90
	#Test	8,816	7,830	5,406	89
	Total	14,697	14,675	12,973	10
NAMODD III	#Train	13,758	13,785	12,275	10
WN18RR-HL	#Valid	465	619	613	9
	#Test	474	623	619	8
	Total	366,079	182,274	95,788	29
VACO2 10 III	#Train	362,728	181,196	95,432	29
YAGO3-10-HL	#Valid	1,662	2,316	2,113	13
	#Test	1,689	2,359	2,135	14

Table 4: Statistics of the filtered sparser datasets.

				FB1	5k-237						
	Subsamp	oling	MR	R	H@	1	Н@	3	Н@	10	
Model	Assumption	Loss	Mean	SD	Mean	SD	Mean	SD	Mean	SD	γ
		NS	23.9	0.2	15.8	0.1	26.1	0.3	40.0	0.2	-
	None	SANS	22.3	0.1	13.8	0.1	24.2	0.0	39.5	0.2	-
		TANS	32.8	0.2	23.2	0.1	36.2	0.2	52.2	0.1	-2
		NS	27.2	0.1	19.1	0.1	29.5	0.1	43.0	0.2	-
	Base	SANS	32.3	0.0	23.0	0.1	35.4	0.1	51.2	0.1	-
ComplEx		TANS	†33.3	0.0	†23.8	0.1	†36.9	0.1	†52.7	0.0	-1
		NS	25.1	0.2	17.1	0.3	27.4	0.2	41.0	0.2	-
	Freq	SANS	32.7	0.1	23.6	0.1	36.0	0.1	51.2	0.1	-
		TANS	†33.3	0.0	†23.8	0.0	36.8	0.1	52.1	0.2	-0.5
		NS	22.8	0.4	14.7	0.5	24.7	0.4	39.0	0.1	-
	Uniq	SANS	32.6	0.0	23.5	0.1	35.8	0.1	51.2	0.1	-
		TANS	33.0	0.1	23.5	0.1	36.5	0.1	52.1	0.1	-0.5
		NS	23.3	0.1	15.6	0.1	25.7	0.1	38.4	0.1	-
	None	SANS	22.3	0.1	14.0	0.2	24.1	0.1	39.2	0.0	-
		TANS	31.0	0.1	21.7	0.1	34.0	0.1	49.6	0.1	-1
		NS	25.4	0.1	17.9	0.1	27.6	0.1	40.4	0.1	-
	Base	SANS	30.8	0.1	21.9	0.1	33.6	0.1	48.4	0.1	-
DistMult		TANS	†31.5	0.1	[†] 22.4	0.1	†34.6	0.1	† 49.7	0.0	-0.:
		NS	24.0	0.1	16.7	0.2	25.9	0.1	38.4	0.1	-
	Freq	SANS	29.9	0.0	21.2	0.1	32.8	0.0	47.5	0.1	-
		TANS	30.7	0.0	21.6	0.0	34.0	0.0	49.0	0.0	-1
		NS	21.0	0.1	13.5	0.2	22.8	0.2	36.3	0.2	
	Uniq	SANS	29.2	0.0	20.5	0.1	31.9	0.0	46.7	0.0	-
	4	TANS	30.7	0.1	21.5	0.1	33.8	0.1	49.3	0.1	-2
		NS	30.4	0.0	21.3	0.1	33.4	0.1	48.5	0.0	-
	None	SANS	33.0	0.1	22.9	0.1	37.2	0.1	†53.0	0.1	-
		TANS	33.6	0.0	23.9	0.0	37.3	0.0	[†] 53.0	0.1	-0.
		NS	29.4	0.0	20.0	0.0	32.8	0.0	48.1	0.0	-0
	Base	SANS	33.0	0.1	23.1	0.1	36.8	0.0	52.7	0.0	
TransE											
TrunsL		TANS	33.0	0.0	23.1	0.0	36.8	0.1	52.7	0.1	-0.
	F	NS	29.3	0.1	20.0	0.1	32.8	0.1	47.8	0.1	-
	Freq	TANS	33.5	0.0	23.9	0.1	37.2	0.1	52.8 52.8	0.1	
						0.1	37.2			0.1	-0.
		NS	30.1	0.1	21.0	0.1	33.6	0.0	48.0 52.7	0.0	-
	Uniq	SANS	33.5		23.9		37.3			0.1	
		TANS	₹34.0	0.1	T24.5	0.1	₹37.7	0.1	₹53.0	0.1	0.5
	None	NS	30.3	0.0	21.4	0.1	33.2	0.1	48.4	0.1	-
		SANS	32.9	0.1	22.8	0.1	36.8	0.0	53.1	0.2	-
		TANS	34.1	0.1	24.6	0.1	37.7	0.1	†53.3	0.1	-0.
	Base	NS	29.5	0.0	20.3	0.0	32.7	0.1	47.9	0.0	-
		SANS	33.6	0.1	23.9	0.1	37.3	0.1	53.1	0.0	-
RotatE		TANS	33.8	0.0	24.2	0.0	37.4	0.0	53.0	0.1	-0.
		NS	29.4	0.1	20.2	0.1	32.6	0.1	47.6	0.1	-
	Freq	SANS	34.0	0.1	24.6	0.0	37.7	0.0	53.0	0.0	-
		TANS	34.1	0.0	24.6	0.0	37.7	0.0	53.1	0.1	-0.0
		NS	30.1	0.0	21.2	0.1	33.3	0.1	47.7	0.1	-
	Uniq	SANS	33.9	0.1	24.4	0.1	37.6	0.1	52.9	0.1	-
		TANS	†34.2	0.0	[†] 24.7	0.1	†37.8	0.0	53.1	0.1	0.5
		NS	30.8	0.1	21.8	0.1	33.8	0.1	48.6	0.1	-
	None	SANS	32.8	0.2	22.7	0.3	36.9	0.1	52.8	0.1	-
		TANS	34.4	0.1	24.9	0.1	37.9	0.2	53.6	0.0	-0.
		NS	30.4	0.1	21.6	0.1	33.3	0.1	48.2	0.0	-
	Base	SANS	34.1	0.1	24.4	0.1	37.9	0.1	53.6	0.2	-
HAKE		TANS	34.1	0.0	24.4	0.0	37.9	0.0	53.7	0.0	-0.0
		NS	30.2	0.1	21.5	0.0	33.1	0.0	47.7	0.1	-
	Freq	SANS	34.7	0.0	25.2	0.1	38.2	0.0	53.8	0.1	-
		TANS	34.6	0.0	25.0	0.1	38.2	0.2	53.7	0.1	0.0
		NS	30.7	0.1	22.2	0.1	33.5	0.1	48.0	0.1	-
	Uniq	SANS	34.7	0.1	25.1	0.1	38.3	0.1	53.9	0.1	-
	-	TANS	†34.9	0.0	[†] 25.4	0.0	†38.6	0.1	[†] 54.0	0.1	0.5
		NS	29.1	0.1	20.6	0.1	31.6	0.1	46.3	0.1	-
	None	SANS	34.7	0.2	24.8	0.2	38.5	0.3	54.4	0.2	-
	· · · · · · · · · · · · · · · · · · ·	TANS	35.6	0.1	26.1	0.1	39.4	0.1	54.5	0.1	-1
		NS	28.1	0.1	19.6	0.1	30.9	0.2	45.1	0.2	-
	Base	SANS	35.2	0.2	25.6	0.2	39.0	0.2	54.4	0.3	-
HousE	20	TANS	35.6	0.1	26.1	0.1	39.4	0.2	54.5	0.1	-0.
LLUUDL		NS	27.9	0.1	19.2	0.1	30.7	0.2	45.2	0.1	-
	Freq	SANS	35.9	0.2	26.4	0.2	39.5	0.2	54.7	0.1	-
	1104	TANS	35.8	0.2	26.4	0.2	39.6	0.2	54.7	0.1	-0.0
		NS	28.8	0.1	20.2	0.2	31.9	0.1	45.7	0.0	-
	Uniq								†54.8		
	Omq	SANS	36.1	0.1	†26.7	0.2	39.8	0.1	-	0.2	-
		TANS	₹36.2	0.1	↑26.7	0.2	†39.9	0.1	↑54.8	0.1	0.1

Table 5: Results on FB15k-237.

	Cubsamalina		WN18RR MPP H@1								
Model	Subsamp		MR		H@		H@		H@		. γ
- Inodei	Assumption	Loss	Mean	SD	Mean	SD	Mean	SD	Mean	SD	
		NS	44.5	0.1	38.1	0.2	48.3	0.2	55.5	0.1	-
	None	TANS	45.0 47.3	0.1	41.0	0.1	46.5 49.1	0.3	53.3	0.3	-2
		NS	45.0	0.0	38.9	0.0	48.6	0.1	55.7 55.7	0.1	
		SANS	46.9	0.1	42.7	0.1	48.5	0.2	55.5	0.1	
~	Base			0.1		0.2	49.3	0.2			-2
ComplEx		TANS NS	47.7	0.2	43.6	0.1		0.2	55.9	0.3	-2
	_	SANS	45.1 47.4	0.1	38.9 43.2	0.1	48.8	0.2	56.0 56.0	0.2	
	Freq										
		TANS	48.0	0.1	43.9	0.1	149.7	0.1	56.1	0.1	-2
		NS	45.0	0.1	38.7	0.1	48.8	0.1	56.0	0.3	-
	Uniq	SANS	47.5	0.1	43.3	0.1	49.1	0.2	56.2	0.2	-
		TANS	†48.3	0.1	†44.4	0.2	49.6	0.1	†56.3	0.2	-]
		NS	38.5	0.2	30.6	0.3	42.9	0.2	52.5	0.1	-
	None	SANS	42.4	0.0	38.2	0.1	43.7	0.0	51.0	0.2	-
		TANS	44.2	0.1	40.1	0.1	45.3	0.1	53.2	0.2	-2
		NS	39.3	0.2	31.9	0.2	43.3	0.1	53.0	0.2	-
	Base	SANS	43.9	0.1	39.4	0.1	45.2	0.1	53.3	0.2	-
DistMult		TANS	44.6	0.0	40.5	0.2	45.7	0.1	53.9	0.1	-2
		NS	39.0	0.2	31.2	0.2	43.2	0.1	52.9	0.2	-
	Freq	SANS	44.5	0.1	40.0	0.1	46.0	0.1	54.2	0.2	-
	•	TANS	44.7	0.1	40.5	0.2	45.8	0.0	54.0	0.2	-2
		NS	38.8	0.2	30.8	0.2	43.1	0.1	53.0	0.2	-
	Uniq	SANS	44.7	0.1	40.1	0.1	†46.2	0.3	54.3	0.0	
	- 1	TANS	†45.0	0.1	†40.7	0.1	46.1	0.2	†54.5	0.2	-0
		NS	21.1	0.0	2.1	0.1	36.5	0.2	50.4	0.2	-0
	3.7		22.5	0.0	1.7	0.1	40.2	0.2	52.5	0.2	
	None	TANS									-
			22.7	0.0	2.5	0.0	39.5	0.2	53.4	0.1	0.
	_	NS	20.3	0.1	1.6	0.1	35.1	0.2	49.9	0.2	
	Base	SANS	22.3	0.0	1.3	0.1	40.2	0.1	52.9	0.1	-
TransE		TANS	22.4	0.1	1.4	0.1	40.1	0.1	53.0	0.1	0.
	Freq	NS	21.0	0.1	1.8	0.1	36.4	0.2	51.0	0.2	_
		SANS	23.0	0.0	1.9	0.1	40.9	0.2	53.6	0.0	-
		TANS	23.1	0.0	2.1	0.0	†41.0	0.1	53.8	0.0	0.
		NS	21.5	0.1	2.2	0.0	37.2	0.1	51.4	0.2	-
	Uniq	SANS	23.2	0.0	2.3	0.1	40.9	0.2	53.6	0.1	-
		TANS	†23.3	0.1	†3.0	0.0	40.2	0.2	†54.4	0.1	0.
	None	NS	47.0	0.1	42.5	0.2	48.6	0.2	55.8	0.3	-
		SANS	47.2	0.1	42.6	0.1	49.1	0.1	56.7	0.0	_
		TANS	47.3	0.1	42.6	0.1	49.1	0.1	56.7	0.1	-0.
		NS	47.0	0.0	42.2	0.1	48.7	0.1	56.3	0.1	_
	Base	SANS	47.5	0.1	42.7	0.2	49.3	0.1	57.2	0.1	_
RotatE		TANS	47.5	0.1	42.7	0.2	49.3	0.1	57.1	0.1	0.0
		NS	47.1	0.1	42.3	0.1	48.7	0.1	56.4	0.1	-
	Freq	SANS	47.7	0.1	†42.9	0.2	49.6	0.0	57.4	0.1	
		TANS	47.7	0.1	42.8	0.2	49.7	0.1	57.4	0.1	0.
		NS	47.2	0.2	42.7	0.2	48.7	0.1	56.3	0.1	-
	Tinin										
	Uniq	SANS	47.7	0.1	†42.9	0.1	49.6	0.1	57.2	0.1	
		TANS	↑47.8	0.2	42.8	0.3	†49.8	0.1	¹ 57.6	0.1	0.
		NS	48.8	0.1	44.5	0.1	50.5	0.2	57.3	0.1	-
	None	SANS	48.9	0.0	44.5	0.2	50.6	0.3	57.7	0.1	-
		TANS	48.9	0.0	44.4	0.1	50.5	0.3	57.8	0.1	0.0
		NS	49.2	0.0	44.6	0.1	51.1	0.1	57.9	0.2	_
	Base	SANS	49.5	0.1	45.0	0.2	51.2	0.2	58.2	0.2	-
HAKE		TANS	49.5	0.1	45.0	0.2	51.2	0.3	58.4	0.2	0.
		NS	49.3	0.1	44.8	0.1	51.3	0.2	58.0	0.2	-
	Freq	SANS	49.7	0.1	45.2	0.2	51.5	0.1	58.4	0.2	-
		TANS	49.7	0.0	45.2	0.2	51.6	0.3	58.4	0.2	-0.
		NS	49.4	0.2	44.9	0.2	51.3	0.2	57.8	0.2	-
	Uniq	SANS	†49.9	0.0	45.3	0.1	[†] 51.8	0.2	[†] 58.6	0.2	-
		TANS	† 49.9	0.1	†45.4	0.1	[†] 51.8	0.2	58.5	0.2	0.0
		NS	47.4	0.1	41.7	0.1	50.2	0.1	57.3	0.1	-
	None	SANS	49.7	0.1	44.8	0.2	51.5	0.1	59.5	0.1	-
	TONE	TANS	50.2	0.1	45.3	0.1	52.0	0.1	60.0	0.1	-0
		NS	48.1	0.1	42.4	0.1	50.9	0.1	58.5	0.2	-
	Base	SANS	51.2	0.1	46.7	0.1	53.0	0.2	60.3	0.1	
Цонгр	Dase	TANS	51.3	0.1	46.7	0.1	53.0	0.2	60.4	0.1	0.0
HousE		NS	48.1	0.1	42.5	0.2	50.9	0.0	58.5	0.1	
	E										
	Freq	SANS	↑51.4	0.1	↑46.8	0.1	↑53.2	0.3	†60.5	0.1	
		TANS	51.3	0.2	46.7	0.2	53.1	0.3	†60.5	0.1	0.0
		NS	48.1	0.1	42.5	0.1	50.8	0.2	58.1	0.1	-
	Uniq	SANS	51.2	0.2	†46.8	0.2	52.7	0.1	60.1	0.1	-
	oniq	TANS	51.1	0.3	46.7	0.5	52.7	0.1	60.0	0.1	-0

Table 6: Results on WN18RR.

					GO3-10						
Model	Subsamp		MR		H@	1	H@	3	H@		- 0/
Model	Assumption	Loss	Mean	SD	Mean	SD	Mean	SD	Mean	SD	γ
		NS	43.5	0.1	32.8	0.2	49.1	0.2	63.7	0.3	-
	None	SANS	49.6	0.2	39.9	0.1	55.3	0.3	67.3	0.2	-
		TANS	49.6	0.2	40.0	0.2	55.4	0.5	67.2	0.3	-0.05
		NS	44.8	0.1	34.5	0.3	50.0	0.2	64.7	0.2	-
	Base	SANS	49.6	0.3	40.1	0.3	55.2	0.4	67.4	0.3	-
RotatE		TANS	49.5	0.3	40.1	0.3	55.0	0.5	67.3	0.3	-0.05
		NS	44.8	0.2	34.5	0.3	50.0	0.1	64.7	0.2	-
	Freq	SANS	49.9	0.2	40.5	0.3	55.5	0.5	67.4	0.3	-
		TANS	49.9	0.2	40.5	0.3	55.5	0.5	67.4	0.2	0.01
		NS	44.4	0.2	34.0	0.3	49.8	0.2	64.3	0.2	-
	Uniq	SANS	50.0	0.3	40.6	0.2	55.6	0.3	67.5	0.2	-
		TANS	[†] 50.1	0.2	† 40.7	0.1	[†] 55.7	0.3	† 67.6	0.3	0.05
		NS	47.4	0.3	36.6	0.5	53.9	0.1	67.0	0.1	-
	None	SANS	53.5	0.2	44.6	0.3	59.1	0.4	69.0	0.2	-
		TANS	53.7	0.1	45.3	0.3	59.0	0.1	68.8	0.1	0.05
		NS	48.8	0.3	38.4	0.4	55.0	0.2	68.1	0.3	-
	Base	SANS	54.6	0.2	46.2	0.3	59.9	0.2	69.6	0.2	-
HAKE		TANS	54.5	0.2	45.9	0.3	59.9	0.2	69.9	0.1	-0.1
		NS	49.3	0.2	39.1	0.3	55.4	0.1	68.1	0.2	-
	Freq	SANS	54.6	0.4	46.0	0.7	60.2	0.1	69.6	0.3	-
	-	TANS	54.8	0.2	46.4	0.3	60.1	0.1	69.6	0.3	0.05
		NS	45.2	0.1	34.3	0.1	51.1	0.1	65.8	0.3	-
	Uniq	SANS	[†] 55.2	0.3	[†] 46.8	0.5	[†] 60.5	0.2	[†] 70.0	0.3	-
		TANS	55.1	0.2	† 46.8	0.3	60.3	0.1	69.9	0.2	-0.1
		NS	29.2	0.0	18.3	0.1	33.6	0.2	50.1	0.2	-
	None	SANS	54.8	1.3	46.8	1.3	59.7	1.2	68.9	1.2	-
		TANS	54.8	1.2	46.9	1.2	59.6	1.2	68.8	1.1	0.01
		NS	29.6	0.1	19.8	0.1	33.6	0.2	48.9	0.1	-
	Base	SANS	56.7	0.1	48.6	0.2	61.7	0.2	71.3	0.1	-
HousE		TANS	57.0	0.2	49.0	0.4	61.9	0.3	[†] 71.5	0.2	-0.1
		NS	27.3	0.8	17.5	0.9	31.0	0.8	46.6	0.8	-
		SANS	57.0	0.1	49.0	0.2	62.0	0.1	71.4	0.1	-
		TANS	57.2	0.1	49.3	0.1	† 62.3	0.1	71.4	0.1	-0.1
		NS	28.1	0.2	18.2	0.4	31.8	0.1	47.6	0.0	-
	Uniq	SANS	57.2	0.1	49.3	0.2	62.0	0.0	71.4	0.2	-
	•	TANS	[†] 57.3	0.2	[†] 49.5	0.3	62.2	0.1	[†] 71.5	0.1	-0.05

Table 7: Results on YAGO3-10.

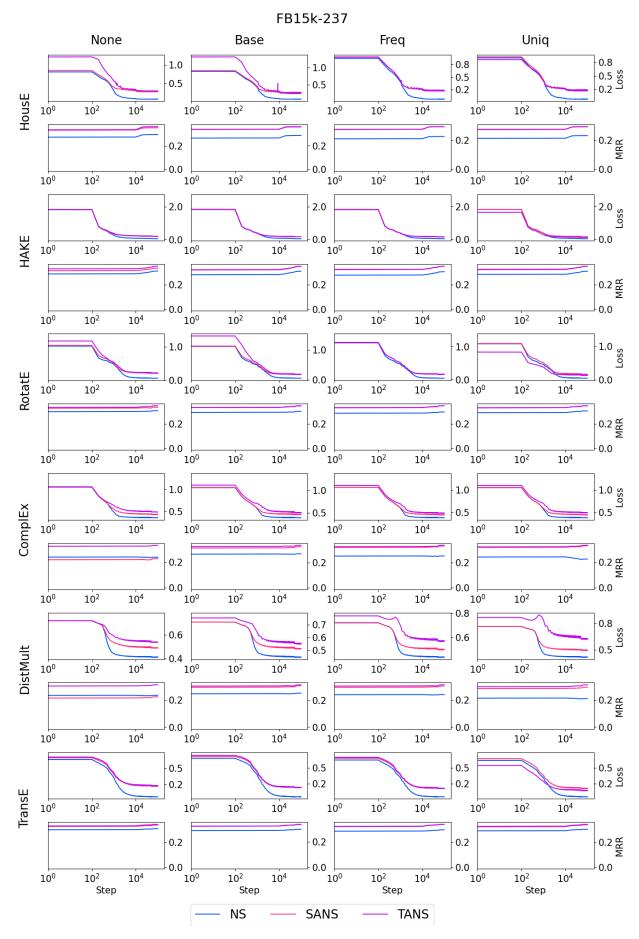


Figure 6: Training loss and validation MRR Curve on FB15k-237.

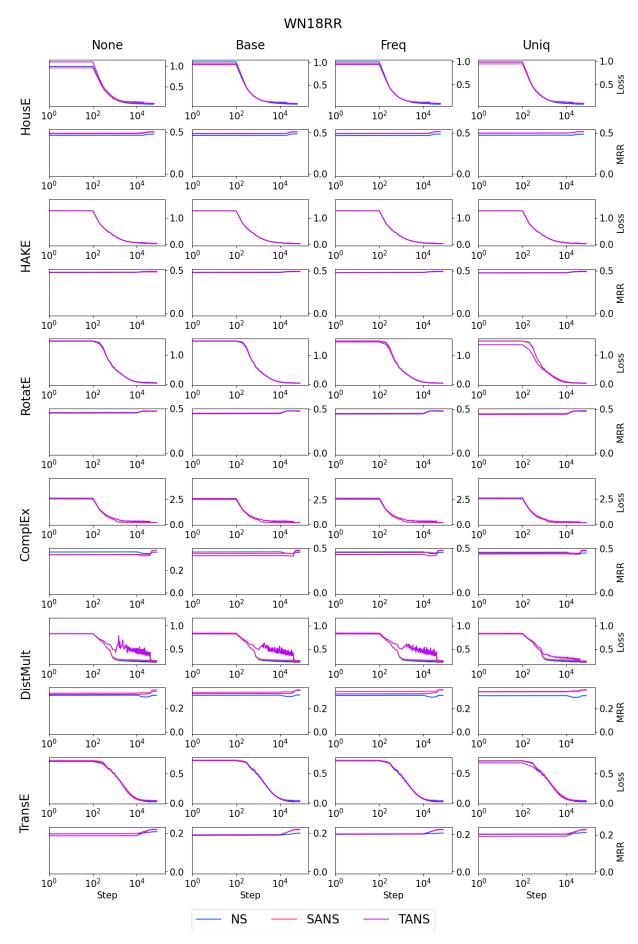


Figure 7: Training loss and validation MRR Curve on WN18RR.

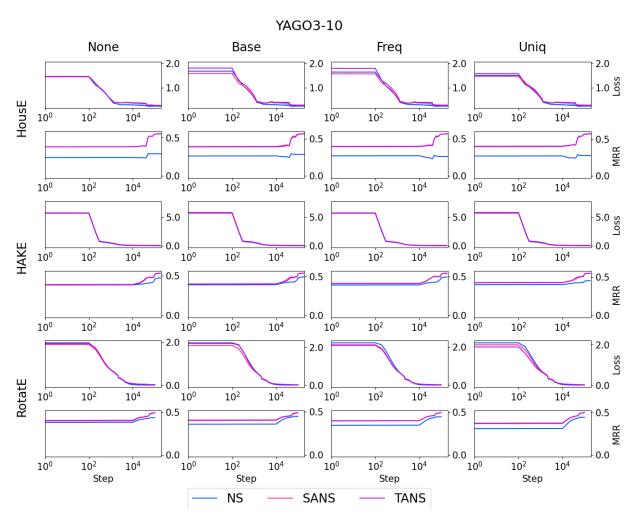


Figure 8: Training loss and validation MRR Curve on YAGO3-10.

		FB151	k-237-H	L			
36.11	Subsamp	ling	MR	R	Н@	1	_
Model	Assumption	Loss	Mean	SD	Mean	SD	γ
		NS	38.1	0.3	28.4	0.5	-
	None	SANS	35.2	0.2	24.5	0.3	-
		TANS	41.1	0.1	33.0	0.1	-1
		NS	40.5	0.1	31.8	0.2	-
	Base	SANS	38.4	0.2	28.9	0.2	-
		TANS	41.8	0.1	33.6	0.2	-1
HAKE		NS	41.1	0.1	32.8	0.1	-
	Freq	SANS	40.2	0.0	31.5	0.1	-
	•	TANS	[†] 42.0	0.1	†33.7	0.1	-1
		NS	41.5	0.1	33.2	0.1	-
	Uniq _	SANS	41.1	0.0	32.8	0.0	-
		TANS	41.9	0.2	33.5	0.2	-0.1
		NS	40.0	0.1	30.8	0.1	-
	None	SANS	36.3	0.1	25.3	0.2	-
		TANS	41.5	0.0	33.1	0.1	-1
		NS	41.8	0.1	33.6	0.1	-
	Base	SANS	40.7	0.1	31.7	0.2	-
		TANS	42.0	0.1	33.8	0.1	-0.5
RotatE		NS	41.3	0.1	33.2	0.1	-
	Freq	SANS	42.0	0.2	33.6	0.3	-
	-1 ,	TANS	†42.3	0.0	† 34.1	0.1	-0.5
		NS	41.7	0.1	33.7	0.2	-
	Uniq	SANS	42.2	0.1	33.8	0.2	-
	•	TANS	42.1	0.1	33.8	0.2	-0.05
		NS	39.1	0.2	29.8	0.2	-
	None	SANS	37.0	0.2	26.2	0.4	-
		TANS	42.3	0.1	34.1	0.2	-2
		NS	40.3	0.1	31.3	0.2	-
	Base	SANS	40.5	0.4	31.3	0.4	-
		TANS	42.4	0.2	34.2	0.3	-2
HousE		NS	39.8	0.3	31.0	0.3	-
	Freq	SANS	42.1	0.2	33.8	0.2	-
	1	TANS	†42.8	0.3	† 34.8	0.4	-1
		NS	40.5	0.2	31.9	0.2	-
	Uniq	SANS	42.4	0.2	34.4	0.2	-
	o.nq	TANS	42.5	0.1	34.5	0.0	-1

	Subsamp	lina	MR	D	Н@	1	
Model							γ
	Assumption	Loss	Mean	SD	Mean	SD	
		NS	10.8	0.1	8.7	0.2	-
	None	SANS	10.3	0.1	7.8	0.1	-
		TANS	13.9	0.2	†12.1	0.2	-2
		NS	12.1	0.2	9.5	0.3	-
	Base	SANS	11.1	0.1	9.1	0.1	-
HARE		TANS	13.7	0.1	11.7	0.3	-2
HAKE		NS	12.4	0.1	10.4	0.1	-
	Freq	SANS	11.9	0.2	9.5	0.2	-
		TANS	†14.2	0.5	11.9	0.4	-2
		NS	13.3	0.3	11.3	0.3	-
	Uniq	SANS	11.9	0.2	9.7	0.2	-
		TANS	14.1	0.2	11.7	0.2	-2
		NS	14.2	0.2	11.8	0.3	-
	None	SANS	13.9	0.3	11.7	0.3	-
		TANS	14.4	0.1	11.8	0.2	-2
		NS	13.9	0.2	11.5	0.2	-
	Base	SANS	14.1	0.3	11.7	0.3	-
		TANS	14.5	0.1	11.7	0.1	-2
RotatE		NS	14.4	0.1	12.0	0.1	-
	Freq	SANS	14.3	0.4	12.0	0.3	-
		TANS	†15.1	0.1	12.2	0.1	-2
		NS	14.4	0.2	12.2	0.1	-
	Uniq	SANS	14.2	0.2	11.9	0.2	-
RotatE		TANS	[†] 15.1	0.2	†12.3	0.3	-2
		NS	10.7	1.8	8.4	1.4	-
	None	SANS	11.7	1.1	9.5	0.9	-
	110110	TANS	13.4	0.4	11.0	0.4	-2
		NS	9.9	0.4	8.4	0.4	-
	Base	SANS	11.5	0.2	9.5	0.2	-
	2400	TANS	13.4	0.2	11.3	0.3	-2
HousE		NS	†13.9	0.1	11.8	0.2	-
	Freq	SANS	13.8	0.2	11.9	0.3	-
	1104	TANS	†13.9	0.3	†12.0	0.2	0.1
		NS	13.7	0.1	11.6	0.1	-
	Uniq	SANS	13.8	0.2	11.6	0.2	_
	Omq	TANS	13.8	0.2	11.7	0.3	-0.0

Table 8: Results on FB15k-237-HL.

Table 9: Results on WN18RR-HL.

	Subsamp	ling	MR	R	H@	1	
Model	Assumption	Loss	Mean	SD	Mean	SD	γ
		NS	45.9	0.0	36.9	0.1	_
	None	SANS	47.8	0.4	40.0	0.6	_
	None	TANS	49.2	0.4	39.8	0.7	-0.5
		NS	50.2	0.3	43.0	0.3	_
	Base	SANS	47.7	0.4	40.5	0.7	_
	Base	TANS	50.1	0.3	41.4	0.3	-0.5
HAKE		NS	[†] 50.8	0.3	†43.3	0.2	_
	Freq	SANS	48.8	0.1	41.3	0.2	_
	Ticq	TANS	49.7	0.3	41.0	0.2	-0.5
		NS	49.4	0.2	40.8	0.2	_
	Uniq	SANS	46.9	0.4	39.8	0.5	
	Omq	TANS	49.4	0.6	40.6	0.8	-0.5
		NS	38.0	0.1	28.7	0.3	_
	None	SANS	41.3	0.1	32.3	0.2	_
	Tione	TANS	43.5	0.1	34.8	0.2	-0.5
		NS	40.6	0.2	31.8	0.5	_
	Base	SANS	43.8	0.2	35.1	0.1	_
	Dasc	TANS	43.8	0.2	35.2	0.1	-0.0
RotatE		NS	40.3	0.2	31.4	0.4	_
	Freq	SANS	43.5	0.2	34.6	0.1	_
		TANS	43.7	0.0	35.1	0.1	-0.1
		NS	40.2	0.0	31.3	0.2	_
	Uniq	SANS	43.9	0.1	35.1	0.2	_
	omq	TANS	[†] 44.1	0.1	[†] 35.4	0.3	-0.1
		NS	37.8	0.3	26.9	0.4	-
	None	SANS	50.3	0.1	40.7	0.3	-
	Tione	TANS	[†] 52.5	0.5	[†] 45.4	0.3	-0.5
		NS	42.8	1.2	34.3	1.9	-
	Base	SANS	51.9	0.3	44.4	0.2	-
	2400	TANS	51.9	0.6	44.3	0.8	0.0
HousE		NS	39.7	0.8	29.9	1.5	-
	Freq	SANS	48.6	1.7	40.0	1.4	-
	1	TANS	52.0	0.1	44.5	0.3	-1
		NS	41.0	0.1	31.6	0.1	-
	Uniq	SANS	49.4	0.3	41.1	1.1	-
	1	TANS	52.2	0.1	44.7	0.1	-0.0

Table 10: Results on YAGO3-10-HL.