

Graph Reasoning Paradigm: Structured and Symbolic Reasoning with Topology-Aware Reinforcement Learning for Large Language Models

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Abstract

Long Chain-of-Thought (LCoT), achieved by Reinforcement Learning with Verifiable Rewards (RLVR), has proven effective in enhancing the reasoning capabilities of Large Language Models (LLMs). However, reasoning in current LLMs is primarily generated as plain text, where performing semantic evaluation on such unstructured data creates a computational bottleneck during training. Despite RLVR-based optimization, existing methods still suffer from coarse-grained supervision, reward hacking, high training costs, and poor generalization. To address these issues, we propose the Graph Reasoning Paradigm (GRP), which realizes structured and symbolic reasoning, implemented via graph-structured representations with step-level cognitive labels. Building upon GRP, we further design Process-Aware Stratified Clipping Group Relative Policy Optimization (PASC-GRPO), which leverages structured evaluation to replace semantic evaluation, achieves process-aware verification through graph-structured outcome rewards, and mitigates reward hacking via stratified clipping advantage estimation. Experiments demonstrate significant improvements across mathematical reasoning and code generation tasks. Data, models, and code will be released later.

1 Introduction

Long chain-of-thought (LCoT) has been proven effective for eliciting the reasoning capabilities of Large Language Models (LLMs), particularly when combined with reinforcement learning with verifiable rewards (RLVR) (El-Kishky, 2024; Chen et al., 2025; He et al., 2025b). Such approaches have been applied to mathematical reasoning (Zhou et al., 2024; Moshkov et al., 2025; He et al., 2025a) and code generation (El-Kishky et al., 2025), which are representative complex reasoning tasks and have become common benchmarks for evaluating the reasoning abilities of LLMs.

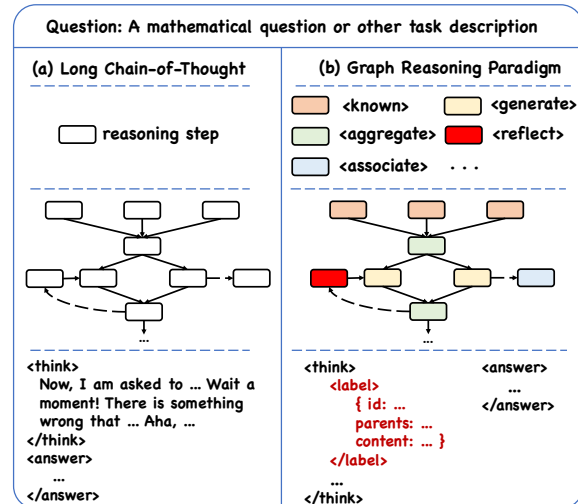


Figure 1: Comparison between traditional Long Chain-of-Thought and Graph Reasoning Paradigm.

However, current LLMs predominantly produce plain-text outputs. Although complex reasoning behaviors such as reflection and Aha moments have emerged (Yang et al., 2025b), the reliability of the reasoning process remains limited (Shojaee et al., 2025). Recently, several reinforcement learning (RL) approaches have attempted to address this issue. However, outcome-based reward methods provide only coarse-grained supervision, offering little semantic control over the reasoning process (DeepSeek-AI et al., 2025). Although process-based reward methods offer semantic control over the reasoning process, they often suffer from high training costs and poor generalization (Yuan et al., 2024). In summary, semantic evaluation is inherently difficult. Since LLMs output plain text, we are forced to perform semantic assessments, which creates a vicious cycle.

Inspired by the structured cognitive mechanisms of human cognition (George et al., 2021), we propose to replace plain-text with structured and symbolic representations, and to replace semantic eval-

uation with structure-based evaluation, thereby enabling low-cost and fine-grained control over the reasoning process when computing outcome rewards. Notably, increasing reliance on process-based rewards may encourage models to over-optimize intermediate reasoning quality at the expense of correct final answers, giving rise to reward hacking (Denison et al., 2024). This issue fundamentally reflects the limitations of conventional advantage estimation methods.

In this paper, we propose a **Graph Reasoning Paradigm (GRP)**, which realizes the structured and symbolic reasoning process using graph-structured representations. As shown in Figure 1, the reasoning process in this paradigm is explicitly organized into a structured form, where each reasoning step is annotated with a specific cognitive label. These labels correspond to different cognitive operations, including *known*, *generate*, *aggregate*, *reflect*, *refine*, *reverse*, and *associate*. This structured representation transforms reasoning from unstructured text into an explicit graph, improving interpretability, and enabling systematic evaluation of reasoning processes. Based on this paradigm, we construct over 40.3k graph-structured mathematical reasoning chains and 12k graph-structured code generation chains with step-level labels, and perform supervised fine-tuning (SFT) to train LLMs to internalize graph-structured reasoning.

Building on the structured and symbolic outputs, we propose **PASC-GRPO**, a **Process-Aware Stratified Clipping** extension of **Group Relative Policy Optimization** to further exert finer control over the reasoning process. We design a set of graph-based outcome rewards that evaluate the quality of the reasoning process using structural properties such as label validity, reachability, connectivity, informative subgraph, and reverse search consistency. These rewards do not rely on value models for semantic evaluation, leading to better generalization and improved training efficiency. To mitigate reward hacking introduced by multiple rewards, we further propose stratified clipping advantage estimation, which separately normalizes rewards within correct and incorrect groups, ensuring stable and reliable optimization.

Experiments across mathematical reasoning and code generation benchmarks demonstrate that our method consistently outperforms strong baselines, particularly on competition-level tasks. Furthermore, it effectively mitigates reward hacking and reduces reasoning length, improving both accuracy

and inference efficiency. In summary, our contributions are threefold:

- We propose a **Graph Reasoning Paradigm** that enables structured and symbolic reasoning processes. We construct over **52k** graph-structured reasoning chains with step-level cognitive labels for SFT.
- We propose **PASC-GRPO**, a reinforcement learning method that leverages graph-structured outcome rewards and stratified clipping advantage estimation to improve reasoning quality while mitigating reward hacking.
- We empirically demonstrate that our approach significantly improves the reasoning performance of LLMs, providing evidence for the effectiveness of graph reasoning paradigm.

2 Related Work

Structured cognition. Structured reasoning frameworks organize the reasoning process into explicit and manageable representations. Examples include Tree-of-Thoughts (Yao et al., 2023), Graph-of-Thoughts (Besta et al., 2023), and cognitive-architecture-inspired models (Sumers et al., 2023). However, these approaches primarily focus on prompting strategies, and rely on semantic evaluation or manual annotation (Chen et al., 2024).

RL-enhanced Reasoning. Reinforcement learning has been widely applied to improve the reasoning capabilities of LLMs (Xu et al., 2025; Zhang et al., 2025; Yuan et al., 2024). However, these methods often rely on coarse reward signals, making fine-grained process control and evaluation difficult, and increasing the risk of reward hacking (Denison et al., 2024). Detailed technical background is provided in Appendix A.

3 Methodology

As illustrated in Figure 2, our method consists of two main components:

- **Graph Reasoning Paradigm**, which enables LLMs to produce symbolized, interpretable, verifiable, and accurate graph-structured reasoning through supervised fine-tuning;
- **PASC-GRPO**, a reinforcement learning framework that incorporates process-aware graph rewards and stratified clipping advantage estimation to further improve reasoning quality and training stability.

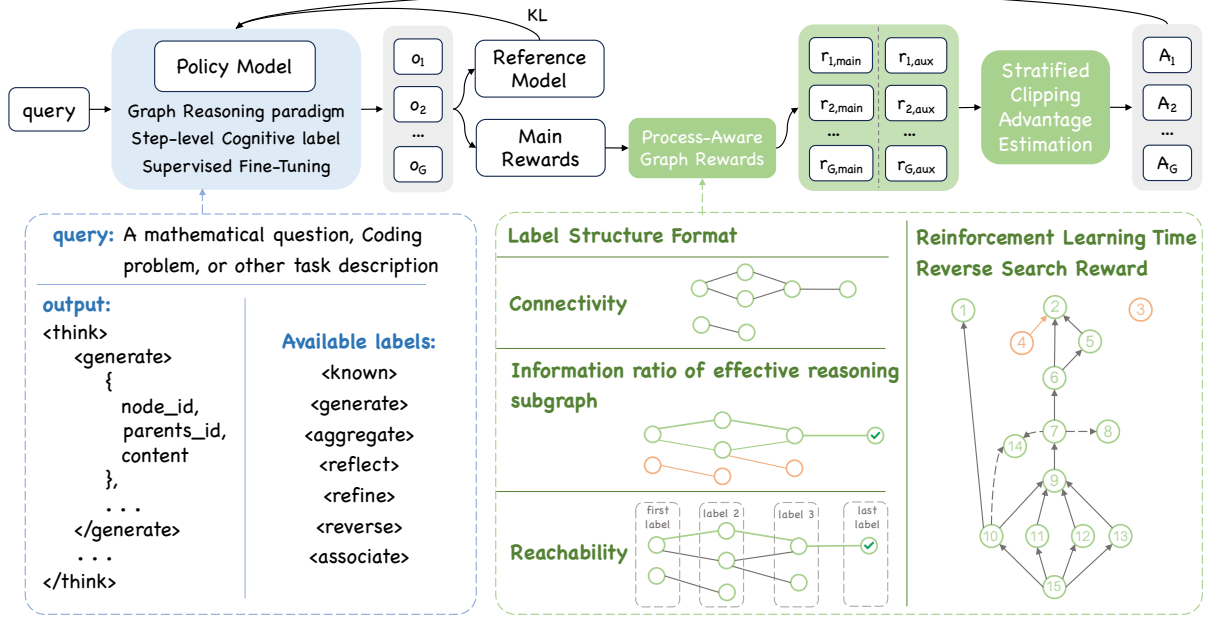


Figure 2: Overview of the proposed method.

3.1 Graph Reasoning Paradigm

We design a graph-structured reasoning paradigm in which the model directly outputs its reasoning process as a graph. This paradigm is realized through step-level cognitive labels, node-based reasoning representation, and data synthesis followed by supervised fine-tuning (SFT).

3.1.1 Step-level Cognitive Labels

Existing large reasoning models employ only coarse-grained `<think>` and `<answer>` tags, without further decomposition of the reasoning process. As a result, emergent operations such as reflection may occur at arbitrary stages and with inconsistent content during reasoning. To address this limitation, we introduce step-level reasoning tags. Details are provided in Appendix B.

By explicitly selecting a cognitive label at each step, the model is encouraged to reason about *how* to emerge, not only *can* emerge, which improves the interpretability of the reasoning process and the reliability of utilizing different reasoning patterns.

3.1.2 Node-based Reasoning Representation

Under the graph reasoning paradigm, each cognitive label wraps one or more reasoning nodes. Each node is defined as a tuple:

$$v = (id, parents_id, content), \quad (1)$$

where *id* is the unique identifier of the node, *parents_id* is a list of parent node ids, and *content*

denotes the content of the reasoning step.

This node-based representation enables the reasoning process to be abstracted into a directed graph, which forms the foundation for graph-structured, process-aware optimization.

3.1.3 Data Synthesis and SFT

To enhance the model’s structural reasoning while maintaining its inherent inference fluidity, we implement an iterative synthesis pipeline. This process consists of three primary stages:

Generating and Validating Reasoning Traces.

For each problem p in the dataset \mathcal{P} , we first sample a reasoning trace using a teacher LLM \mathcal{T} and verify its final answer against the ground truth. If the answer is correct, the trace is retained for subsequent graph-structured transformation. If the answer is incorrect, we trigger a regeneration process with an upper bound on the number of retries. This regeneration continues until either a correct reasoning trace is obtained or the maximum retry limit is reached. Only reasoning traces with verified correct answers are passed to the next stage.

Graph-Structured Translation. Given a validated reasoning trace T , we translate it into the Graph Reasoning Paradigm. This translation explicitly exposes the latent reasoning structure while preserving the original semantic content of the trace. Graph representations derived from originally correct and regenerated traces are jointly col-

lected for quality inspection. Detailed prompts and examples are provided in Appendix C.

Graphical CoT Verification and Refinement.

To ensure the quality of the synthesized graph-structured reasoning data, we introduce an automated quality control loop that evaluates each generated graph from two complementary aspects:

- **Node–Label Consistency.** We verify that the reasoning content of each node conforms to the semantic scope of its assigned label.
- **Parent–Child Coherence.** We check whether each node is logically coherent with its parent node(s), ensuring a consistent and non-contradictory reasoning flow.

If a graph G fails either of the above checks, we generate structured feedback that explicitly identifies the problematic nodes and the corresponding violation types. This feedback, together with a pre-defined refinement prompt template, is fed back to the LLM to trigger a re-translation step. The refinement process iterates until the graph passes all quality checks or a maximum number of translation attempts is reached.

Detailed prompts are provided in Appendix D. Detailed statistics on the data synthesis process are provided in the Appendix E. The cases are provided in Appendix F.

Supervised Fine-Tuning with Mixed Reasoning Formats.

After constructing the graph-structured reasoning dataset, we perform supervised fine-tuning on both graph-structured traces and a subset of original CoT traces that could not be converted into valid graphs. Although these samples do not conform to the graph format, their problem–solution pairs and coherent reasoning steps remain informative. This mixed-format training allows the model to internalize the graph reasoning paradigm while preserving fluency, completeness, and generalization of its native reasoning behavior.

3.2 PASC-GRPO

Fine-grained control over the reasoning process is crucial for solving complex reasoning tasks. (Shao et al., 2025) After SFT, the model internalizes the graph reasoning paradigm, where the reasoning process can be abstracted as a graph independent of specific reasoning content. Building on it, we propose **PASC-GRPO (Process-Aware Stratified Clipping GRPO)**, which enables optimization of reasoning length and process quality.

3.2.1 Process-Aware Graph Rewards

We use NetworkX (Hagberg et al., 2008) to construct reasoning graphs. On this basis, we design *Process-Aware Graph Rewards*, which can be grouped into three categories:

- **Graph structure format rewards**, including the *Label Structure Format Reward*, which enforce valid and well-formed reasoning graphs.
- **Reasoning length rewards**, including the *Connectivity Reward*, which encourages fewer connected components, and the *Information Ratio of Effective Reasoning Subgraph*, which promotes shorter reasoning paths within each component.
- **Reasoning process quality rewards**, including the *Reachability Reward* at the global process level, and the *Reverse Search Reward* at the step level.

Label Structure Format Rewards. We introduce it to enforce the structural validity of graph-structured reasoning. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote the reasoning graph, where $Pa(v)$ is the set of parent nodes of v . Let \mathcal{K} , \mathcal{A} , and \mathcal{R} denote *known*, *aggregate*, and *refine* nodes, and \mathcal{T} denote the set of reasoning tags. The overall format reward R_{fmt} is defined as the average of three complementary sub-rewards:

$$R_{fmt} = \frac{1}{3} (R_{dens} + R_{topo} + R_{para}). \quad (2)$$

- **Node Density.** For *aggregate* and *refine*, each tag is required to wrap exactly one node:

$$R_{dens} = \frac{1}{|\mathcal{A} \cup \mathcal{R}|} \sum_{v \in \mathcal{A} \cup \mathcal{R}} \mathbb{1}(|v|_{\text{label}} = 1). \quad (3)$$

- **Topological Validity.** Specifically, *known* nodes must not depend on prior reasoning, *aggregate* nodes must combine multiple predecessors, and *refine* nodes must extend exactly one preceding step:

$$R_{topo} = \frac{1}{|\mathcal{K} \cup \mathcal{A} \cup \mathcal{R}|} \sum_{v \in \mathcal{K} \cup \mathcal{A} \cup \mathcal{R}} \Phi(v), \quad (4)$$

$$\Phi(v) = \begin{cases} 1, & |Pa(v)| = 0, v \in \mathcal{K}, \\ 1, & |Pa(v)| > 1, v \in \mathcal{A}, \\ 1, & |Pa(v)| = 1, v \in \mathcal{R}, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

- **Parallelism.** Nodes within a tag are treated as parallel reasoning units and are therefore prohibited from forming parent–child relations:

$$R_{para} = \frac{1}{|\mathcal{T}|} \sum_{T \in \mathcal{T}} \mathbb{1} \left(\forall v_i, v_j \in T, v_i \notin Pa(v_j) \right) \quad (6)$$

Connectivity Reward. To discourage fragmented reasoning, we define the connectivity reward based on the number of connected subgraphs:

$$R_{conn} = \frac{1}{n}, \quad (7)$$

where n is the number of connected subgraphs in \mathcal{G} . This reward favors reasoning with fewer isolated branches, resembling more focused reasoning.

Information Ratio of Effective Reasoning Subgraph. We define the effective reasoning subgraph (ERS) as the part of the reasoning graph that effectively contributes to reaching the final answer. The shortest path from the initial conditions to the answer node forms a backbone of effective reasoning. Reasoning branches that diverge from this backbone but eventually merge back are also considered effective. Branches that do not reconnect are treated as ineffective explorations, as they may correspond to incorrect reasoning attempts.

$$\mathcal{V}_{ERS} = \{v \in \mathcal{V} \mid v_{start} \rightsquigarrow v \rightsquigarrow v_{end}\}. \quad (8)$$

The ERS Information Ratio R_{ers} is measured by token count $I(\cdot)$:

$$R_{ent} = \frac{\sum_{v \in \mathcal{V}_{ERS}} I(v)}{\sum_{v \in \mathcal{V}_{Total}} I(v)} \quad (9)$$

This encourages the model to avoid "dead-end" branches and redundant chatter.

Reachability Reward. To mitigate the "wrong process, correct answer" phenomenon (Akter et al., 2025), we evaluate the end-to-end logical flow:

$$R_{reach} = \mathbb{1}(v_{start} \rightsquigarrow v_{end}) \quad (10)$$

where $R_{reach} = 1$ if there exists a directed path in \mathcal{G} , ensuring the reasoning process is complete and leads to the final answer.

Reverse Search Reward. Unlike forward heuristics such as MCTS or PRM, which incrementally expand reasoning paths to approximate the correct answer, our reward directly leverages the known answer. After the reasoning graph \mathcal{G} is completed, we traverse it backward from the answer node v_{ans}

to identify which nodes actually contribute to reaching the answer. For each node $v \in \mathcal{V}$, we assign

$$r(v) = \begin{cases} 1 & \text{if } v \rightsquigarrow v_{ans}, \\ 0 & \text{otherwise,} \end{cases} \quad (11)$$

where $v \rightsquigarrow v_{ans}$ indicates that v is reachable from v_{ans} via backward traversal. The total reward is computed as the average over all nodes:

$$R_{rev} = \frac{1}{|\mathcal{V}|} \sum_{v \in \mathcal{V}} r(v). \quad (12)$$

Overall Process-Aware Graph Reward. The final process-aware graph reward is defined as a weighted sum with scalar weights w_1, \dots, w_5 , where each component reward is normalized to $[0, 1]$ and $\sum_{i=1}^5 w_i = 1$:

$$R_{graph} = w_1 R_{fmt} + w_2 R_{conn} + w_3 R_{ers} + w_4 R_{reach} + w_5 R_{rev} \quad (13)$$

3.2.2 Stratified Clipping Advantage Estimation

To mitigate the reward hacking phenomenon — where the model might optimize graph-structured rewards despite producing incorrect answers — we propose the **Stratified Clipping Advantage Estimation** (SCAE) method. This approach hierarchically prioritizes task accuracy over auxiliary structural signals. The logical flow of the proposed SCAE is illustrated in Figure 3.

Group Stratification. For a sampled group of G reasoning traces, we first calculate the mean accuracy reward $\bar{r}_{acc} \in [0, 1]$. The group is then partitioned into a **Correct Group** (\mathcal{G}_{corr}) where $r_{acc}^{(i)} = 1$ and a **Wrong Group** (\mathcal{G}_{wrong}) where $r_{acc}^{(j)} = 0$. We define the baseline accuracy advantages for these strata as:

$$A_{acc} = \begin{cases} 1 - \bar{r}_{acc}, & \text{if } i \in \mathcal{G}_{corr} \\ 0 - \bar{r}_{acc}, & \text{if } j \in \mathcal{G}_{wrong} \end{cases} \quad (14)$$

This ensures the baseline advantage is non-negative ($A_{acc} \geq 0$) for the correct group and non-positive ($A_{acc} \leq 0$) for the wrong group.

Asymmetrical Auxiliary Clipping. Within each stratum, we calculate the mean of the auxiliary graph rewards, denoted as \bar{r}_{aux}^+ for the correct group and \bar{r}_{aux}^- for the wrong group. Here, the auxiliary reward refers to all rewards other than the main reward (e.g. accuracy reward), such as graph or format rewards. To prevent structural

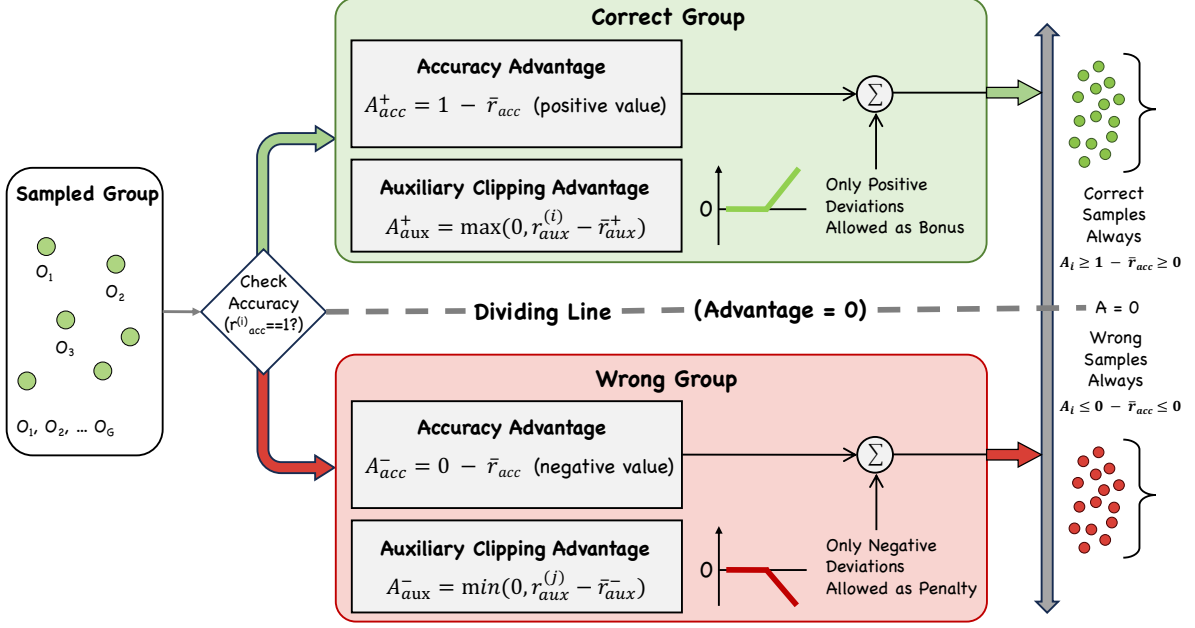


Figure 3: Illustration of the Stratified Clipping Advantage Estimation.

rewards from overriding the accuracy signal, we apply asymmetrical clipping:

- **Correct Group Clipping:** In \mathcal{G}_{corr} , structural rewards are treated strictly as bonuses. Even if a trace’s structural reward $r_{aux}^{(i)}$ is below the group mean, it is not penalized, ensuring the final advantage A_i never falls below the accuracy baseline:

$$A_i = (1 - \bar{r}_{acc}) + \max(0, r_{aux}^{(i)} - \bar{r}_{aux}^+) \quad (15)$$

to ensure $i \in \mathcal{G}_{corr}$, $A_i \geq 1 - \bar{r}_{acc} \geq 0$.

- **Wrong Group Clipping:** In \mathcal{G}_{wrong} , structural rewards serve exclusively as penalties. No matter how high the structural quality $r_{aux}^{(j)}$ is relative to the mean, it receives no credit, ensuring the advantage remains non-positive:

$$A_j = (0 - \bar{r}_{acc}) + \min(0, r_{aux}^{(j)} - \bar{r}_{aux}^-) \quad (16)$$

to ensure $j \in \mathcal{G}_{wrong}$, $A_j \leq 0 - \bar{r}_{acc} \leq 0$.

In summary, SCAE offers the following advantages for training graph-reasoning models:

- **Accuracy Primacy:** By setting a hard floor for correct samples ($A_i \geq A_{acc}^+ \geq 0$) and a hard ceiling for incorrect samples ($A_j \leq A_{acc}^- \leq 0$), SCAE ensures that correctness remains the dominant optimization objective.
- **Reward Hacking Resilience:** The asymmetric clipping mechanism prevents the model

from "cheating" by generating high-quality graph structures for incorrect derivations to offset the accuracy penalty.

4 Experiments

4.1 Experimental Setup

Baselines. We include LLaMA-3.1-8B-Instruct from Meta (Dubey et al., 2024) and Gemma-3-12B-IT / Gemma-3-27B-IT from Google (Kamath et al., 2025) as comparison baselines. To evaluate the effectiveness of GRP and PASC-GRPO, we train on the pre-trained Qwen3-4B-Base and Qwen3-8B-Base models from Qwen (Yang et al., 2025a). Detailed training data construction procedures are provided in Appendix G.

Benchmarks. For mathematical reasoning, the benchmarks includes GSM8K (Cobbe et al., 2021), MATH500 (Lightman et al., 2023), and competition-level benchmarks such as AMC23 (Math AI Dataset Collection, 2025c), AIME24 (Math AI Dataset Collection, 2025a), and AIME25 (Math AI Dataset Collection, 2025b). For code generation, we utilize the widely-used MBPP (Austin et al., 2021), MBPP+ (Liu et al., 2023), HumanEval (Chen et al., 2021), HumanEval+ (Liu et al., 2023), and the highly challenging LiveCodeBench v5 (Jain et al., 2024).

Evaluation Metrics. For larger datasets (GSM8K, MATH500, and LiveCodeBench), we

Table 1: Experimental results on Mathematical Reasoning and Code Generation benchmarks.

Mathematical Reasoning					
Model	GSM8K	MATH500	AMC23	AIME24	AIME25
LLaMA-3.1-8B-Instruct	85.00	54.80	41.20	6.30	2.70
Gemma-3-12B-IT	94.40	85.60	77.30	22.40	18.80
Gemma-3-27B-IT	95.90	90.00	80.50	32.60	24.00
Qwen3-4B-Base	70.20	55.43	19.17	10.00	6.33
+ GRP-SFT	82.30	72.12	63.33	26.67	18.70
+ PASC-GRPO	90.45	85.32	71.43	33.90	24.13
Qwen3-8B-Base	73.01	60.08	38.12	10.00	7.60
+ GRP-SFT	87.72	81.60	75.00	40.00	33.33
+ PASC-GRPO	95.37	91.20	82.50	46.67	38.79
Code Generation					
Model	MBPP	MBPP+	HumanEval	HumanEval+	LiveCodeBench
LLaMA-3.1-8B-Instruct	61.20	52.30	69.70	62.80	10.80
Gemma-3-12B-IT	73.00	62.10	85.40	78.20	25.70
Gemma-3-27B-IT	74.40	63.00	87.80	80.00	26.90
Qwen3-4B-Base	62.40	51.30	75.60	70.70	21.14
+ GRP-SFT	65.43	54.2	81.67	76.90	42.93
+ PASC-GRPO	67.33	55.43	82.45	77.83	46.79
Qwen3-8B-Base	73.27	61.73	80.47	74.27	29.76
+ GRP-SFT	75.45	64.71	86.79	82.09	53.93
+ PASC-GRPO	77.49	65.24	88.43	83.98	56.12

report the avg@3 accuracy; for other benchmarks, we report the avg@16 accuracy. Considering the extensive derivation required for competition-level mathematics (AIME24, AIME25), we set the max_completion_tokens to 32K, while maintaining a limit of 8K for other datasets.

4.2 Main Results

Performance Overview. Table 1 reports the performance of our models on mathematical reasoning and code generation benchmarks. Both GRP-SFT and PASC-GRPO consistently improve accuracy, demonstrating that the proposed GRP effectively enhances reasoning capabilities.

Impact of GRP-SFT. GRP-SFT brings substantial gains in mathematical reasoning. For Qwen3-8B-Base, accuracy improves by 21.52% on MATH500 and 18.30% on GSM8K, with a particularly large gain of 36.88% on the competition-level AMC23 benchmark. Code generation tasks also benefit from GRP-SFT, indicating that the proposed graph-based reasoning paradigm generalizes beyond mathematics.

Effect of PASC-GRPO. PASC-GRPO further enhances performance, especially on high-difficulty benchmarks. On AIME24, Qwen3-4B and Qwen3-8B achieve additional gains of 7.23% and 6.67% over the SFT stage, respectively. These results suggest that the process-aware reward are effective for multi-step problems.

Comparison with Open-Source LLMs. Despite using fewer parameters and training resources, our best model surpasses widely adopted open-source models on competition-level benchmarks. In particular, it surpasses Gemma-3-27B-IT by over 14% accuracy on both AIME24 and AIME25, and achieves comparable or better performance on standard math and code benchmarks such as MATH500, AMC23, MBPP+, and HumanEval+. These results highlight the effectiveness of GRP and PASC-GRPO.

4.3 Ablation Studies

Ablation of GRP and PASC-GRPO. We study the effectiveness of GRP and PASC-GRPO. For SFT, we use the same data as GRP-SFT but replace graph-structured reasoning with standard chain-of-thought. As shown in Table 2, this variant performs

Table 2: Ablation Experiment Results. Accuracy (%).

Method	MATH500	AMC23	AIME24
Qwen3-8B-Base	60.08	38.12	10.00
GRP-SFT	81.60	75.00	40.00
SFT	76.45	67.80	31.50
PASC-GRPO	91.20	82.50	46.67
GRPO	88.75	79.85	43.80
w/o Reachability	87.45	79.30	40.15
w/o Reach. & RSR	83.12	76.55	35.80
w/o SCAE	84.15	76.20	32.50

worse than GRP-SFT on all benchmarks.

For RL, we compare PASC-GRPO with the original GRPO under identical training settings. GRPO consistently yields lower performance. These results show that both graph-structured reasoning and process-aware rewards are essential for strong reasoning performance.

Ablation of Reasoning-Length Rewards. We remove the connectivity reward and further remove ERS Ratio, starting from the GRP-SFT baseline. Figure 4 shows the changes in accuracy and length during training, comparing the cases with and without connectivity and ERS Ratio rewards. For more details on the ablation studies, see Appendix H.

Ablation of Reasoning process quality rewards. Table 2 reports the ablation results of the reachability and reverse search rewards, starting from the GRP-SFT baseline. Removing the reachability reward leads to a performance drop across all benchmarks. Further removing the reverse search reward causes a more substantial degradation. These results indicate that both rewards are critical for guiding effective graph reasoning.

Ablation of SCAE. Table 2 shows that using the original GRPO instead of Stratified Clipping Advantage Estimation (SCAE) consistently degrades performance across all benchmarks. These results suggest that SCAE is crucial for stabilizing the reasoning process and improving accuracy.

4.4 Case Study

In this section, we provide an example to explain how our method solves problems that traditional methods cannot solve. As shown in Figure 5, there are two issues in the below problem-solving process. Firstly, Step 2 is a valid mathematical expression, but it does not contribute to obtaining the final answer. Secondly, there is actually no true

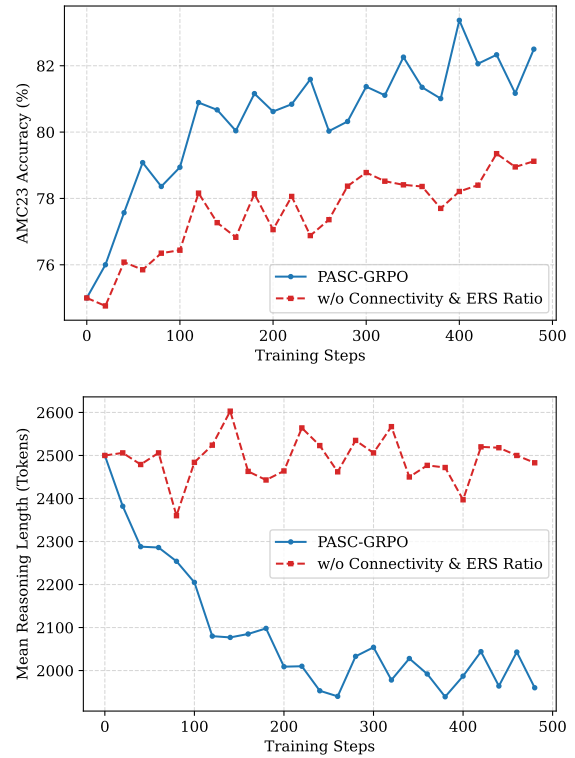


Figure 4: (Top) Accuracy comparison between PASC-GRPO and its variants without connectivity and ERS Ratio rewards on AMC23. (Bottom) Corresponding changes in mean reasoning length during training.

reasoning path from the beginning to obtaining the answer, but step 4 yielded the correct answer.

Standard CoT

Question: Two people have a total of 10 apples, with one person having 2 more than the other. How many apples does the person with more apples have?

Response:

- **Step 1:** Let x be the number of apples.
 $x + y = 10$.
- **Step 2:** Let's try squaring both sides:
 $x^2 + 2xy + y^2 = 100$.
- **Step 3:** Also, $x - y = 2$.
- **Step 4:** Therefore, the answer is $x = 6$.

Figure 5: An example of Standard CoT.

- **Traditional Outcome Reward:** Wrongly assigns a perfect score (1.0) because the final extracted answer is 6, which is correct.

- **Traditional PRM:** Might assign high scores to Steps 1, 2, and 3 because the equations are mathematically sound and locally coherent, failing to realize they don't logically entail Step 4 without missing intermediate steps.

Translation to GRP

```

<think>
  <known>
    {node_id:1
     parent_id:none
     content:Let  $x$  be the number
     of apples.  $x + y = 10$ .}
  </known>
  <generate>
    {node_id:2
     parent_id:1
     content:Let's try squaring
     both sides:  $x^2 + 2xy + y^2 = 100$ .}
  </generate>
  <generate>
    {node_id:3
     parent_id:none
     content:Also,  $x - y = 2$ .}
  </generate>
  <generate>
    {node_id:4
     parent_id:3
     content:Therefore, the answer
     is  $x = 6$ .}
  </generate>
</think>

```

Figure 6: An example of GRP.

As shown in Figure 6, the resulting graph topology is: 1->2(no connection)3->4. By applying our methods, the model can identify the flaws:

- **Reachability Reward:** The graph lacks a continuous path from the <known> to the final answer node. Evaluates to 0.
- **Reverse Search Reward:** When we start at the Answer Node (Node 4) and traverse backward, we cannot reach Nodes 1 and 2. They are identified as “ineffective exploration,” and their RSR scores evaluate to 0.

While standard methods are “fooled” by the correct answer and fluent text, our topology rewards detect that the structural chain of logic is broken.

Table 3: Performance Comparison on Code Generation

Model	MBPP	MBPP+	LiveCodeBech
qwen3-8B	76.70	65.10	57.29
qwen3-8B-Base	73.27	61.73	29.76
+ GRP-SFT	75.45	64.71	53.93
+ PASC-GRPO	77.49	65.24	56.12

4.5 Discovery

As shown in Table 3, our best model trained from Qwen3-8B-Base matches or surpasses the performance of Qwen3-8B on hard code generation benchmarks. Qwen3-8B is a state-of-the-art model distilled using extensive human and computational resources. These results highlight the effectiveness of our methods in handling complex reasoning tasks, especially code generation.

5 Conclusion

In this paper, we propose the Graph Reasoning Paradigm (GRP), which shifts LLM reasoning from plain text to structured symbolic graph representations. We further introduce PASC-GRPO, a reinforcement learning framework that leverages topology-aware rewards and Stratified Clipping Advantage Estimation to enhance reasoning quality while mitigating reward hacking. Our experiments across math and coding demonstrate that GRP significantly improves performance. This work underscores the potential of structured symbolic reasoning and topology-aware rewards in developing more reliable and efficient reasoning models.

Limitations

While the Graph Reasoning Paradigm and PASC-GRPO demonstrate substantial improvements in mathematical reasoning and code generation, several promising directions remain for future exploration. Although we have verified the effectiveness of our approach on 4B and 8B parameter models, investigating the scaling laws of graph reasoning on larger-scale foundation models could reveal further performance gains.

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A Technical Background

A.1 Group Relative Policy Optimization (GRPO)

GRPO (Shao et al., 2024) is a reinforcement learning algorithm designed for training language models with group-based advantage estimation. For each question q , a group of outputs $\{o_1, o_2, \dots, o_G\}$ are sampled from the old policy model $\pi_{\theta_{old}}$ and optimizes the current policy π_θ using:

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}_{q \sim P(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{old}}(\cdot|q)} \left(\frac{1}{G} \sum_{i=1}^G \frac{1}{|o_i|} \sum_{t=1}^{|o_i|} \left(\min \left[r_{i,t}(\theta) \hat{A}_{i,t}, \text{clip}(r_{i,t}(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_{i,t} \right] - \beta \text{D}_{KL}(\pi_\theta \| \pi_{\text{ref}}) \right) \right). \quad (17)$$

$$r_{i,t}(\theta) = \frac{\pi_\theta(o_{i,t} | q, o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t} | q, o_{i,<t})}. \quad (18)$$

$$\hat{A}_{i,t} = \tilde{r}_i = \frac{r_i - \text{mean}(r)}{\text{std}(r)}. \quad (19)$$

A.2 Outcome Rewards and Process Rewards

In reinforcement learning for LLMs, rewards can be categorized into two types based on when they are assigned during the generation process.

Outcome Rewards evaluate the final result of a complete generation sequence. Given a state-action trajectory $(s_1, a_1, \dots, s_T, a_T)$, the outcome reward is defined as $R = r(s_T)$, where only the terminal state s_T receives a reward signal. This approach is commonly used in tasks where quality can only be judged after seeing the complete output, such as code correctness or final answer accuracy.

Process Rewards, on the contrary, provide feedback at each intermediate step of the reasoning process. The cumulative reward is computed as $R = \sum_{t=1}^T r(s_t, a_t)$, where $r(s, a)$ assigns credit to each reasoning step. Process rewards enable more fine-grained supervision and can guide the model toward correct reasoning paths even when the final answer is incorrect.

A.3 Monte Carlo Tree Search (MCTS)

Monte Carlo Tree Search (Xie et al., 2024) is a heuristic search algorithm that builds a search tree by iteratively selecting, expanding, simulating, and backpropagating rewards. In the context of LLM

Step-level Cognitive Labels

- **KNOWN:** known conditions and goals of the problem;
- **GENERATE:** generate reasoning steps that moves the thinking forward
- **AGGREGATE:** merging multiple steps to derive a conclusion;
- **REFINE:** improving an existing step without substantial progress;
- **REFLECT:** reviewing a previous reasoning step;
- **REVERSE:** reasoning backward from the goal to derive a solution strategy;
- **ASSOCIATE:** drawing analogies from similar reasoning structures

Figure 7: Step-level Cognitive Labels.

reasoning, each node represents a partial generation state s , and edges represent token or sequence actions a .

The algorithm balances exploration and exploitation using the Upper Confidence Bound (UCB) formula:

$$\text{UCB}(s, a) = Q(s, a) + c \sqrt{\frac{\ln N(s)}{N(s, a)}}. \quad (20)$$

where $Q(s, a)$ is the estimated value of taking action a in state s , $N(s)$ is the visit count of state s , $N(s, a)$ is the visit count of the state-action pair, and c is the exploration constant.

B Step-level Cognitive Labels

Figure 7 illustrates the step-level cognitive labels used in our Graph Reasoning Paradigm. Each reasoning step in the structured graph is annotated with one of these labels, which represent distinct cognitive operations such as *known*, *generate*, *aggregate*, *reflect*, *refine*, *reverse*, and *associate*. These labels enable fine-grained symbolic abstraction of the reasoning process, facilitating both interpretability and structured evaluation of model behavior.

C Graph-Structured Translation Prompt and Examples

C.1 Graph-Structured Translation Prompt

We design a unified prompt for graph-structured reasoning translation, which specifies the node schema, thinking-mode tags, and dependency relations. The prompt is shown in Figure 10. The illustrative examples mentioned in the prompt are presented in the following subsections. The original Chain-of-Thought traces are generated using Qwen3-Max-Preview, which provides explicit thinking processes, and are subsequently translated into graph-structured representations using the Qwen3-Max model.

C.2 Mathematical Examples

We first present two mathematical examples that are originally included in the graph-structured translation prompt. The first example is designed to explicitly illustrate the use of *reverse thinking* and *associative thinking* (Figure 11). The second example demonstrates another mathematical reasoning process and is accompanied by a schematic visualization of its corresponding reasoning graph, where nodes represent intermediate reasoning steps and edges encode their dependency relations (Figures 12 and 13).

C.3 Code Generation Examples

We include three code generation examples. Each example corresponds to a representative question type of code generation task.

- **Code Completion (Complete).** This type simulates the automatic code completion scenario in integrated development environments (IDEs). The model is provided with a function signature and a docstring as context and is required to generate the remaining function body. This task reflects the core capabilities of base code models such as Codex and StarCoder, primarily evaluating syntactic correctness and context-aware continuation, as exemplified by benchmarks like HumanEval (see Figure 14).
- **Instruction Following (Instruct).** Also referred to as instruction-based or conversational code generation, this paradigm evaluates a model’s alignment with human intent. The input consists of a natural language description of a programming task, and the

model must generate a corresponding functional implementation. This setting is the dominant evaluation protocol for instruction-tuned models such as GPT-4 and DeepSeek-Coder-Instruct, emphasizing the translation of natural language semantics into executable logic (see Figure 15).

- **Competitive Programming (Online Judge).** This type emulates algorithmic contest environments such as ACM-style competitions or online judges (e.g., LeetCode). Unlike instruction-following tasks that generate a single function, competitive programming requires the model to produce a complete script that handles standard input and output streams. As a result, this paradigm is widely regarded as the most challenging form of code generation, demanding advanced algorithmic reasoning as well as robust handling of input parsing, output formatting, and edge cases (see Figure 16).

D Graphical CoT Verification and Refinement Prompt

This appendix presents the prompt for graphical Chain-of-Thought (CoT) verification and refinement, which is shown in Figure 17. All verification and refinement results are generated using the Qwen3-Max model.

E Statistics of the Data Synthesis Process .

Relying on LLMs for structural extraction carries an inherent risk of information distortion. To quantify the robustness of our pipeline, we report the data synthesis statistics in Table 5.

As shown above, a noticeable portion of traces triggered the refinement loop (23.0% for Math, 31.0% for Code). The final success rate of converting these into valid graphs was extremely high. The very small fraction of failures was primarily caused by exceeding the `max_input_length` of the translation model.

F Cases

During translation, we observed two primary types of potential distortion, which our automated quality control loop successfully identifies and corrects, as shown in Figure 8 and Figure 9

Table 4: Ablation Study Results of Length-Reducing Rewards.

Method	MATH500		AMC23		AIME24	
	Acc.	Length	Acc.	Length	Acc.	Length
PASC-GRPO	91.20	1260	82.50	1960	46.67	7040
w/o Connectivity	91.35	1850	81.55	2140	45.10	11200
w/o Conn. & ERS Ratio	90.52	1920	79.12	2483	43.25	22500

Table 5: Statistics of SFT Data Generation.

Dataset	Original Size	Correct Traces	Requiring Refinement	Final Graphs	Success Rate
DeepScaleR	40,315	33,752	7,763 (23.0%)	33,737	99.9%
KodCode	12,000	10,096	3,130 (31.0%)	9,894	98.0%

Case 1: Label Misallocation

During translation, the model occasionally assigns an incorrect cognitive label to a valid reasoning step.

Example:

```
<aggregate>
{
  node_id: 10
  parents: 9
  content: Subtracting equation (i)
           from equation (ii):
           0 = 144s + 288 - st - 2t
           - (240s - st) = -96s + 288 - 2t.
}
</aggregate>
```

Resolution: In this step, a mathematical derivation was incorrectly wrapped in an <aggregate> tag. The label should be <generate>. Our **Node-Label Consistency** check successfully identifies such structural mismatches and triggers refinement to correct the label.

Figure 8: An example of label misallocation.

Case 2: Parent-Child Incoherence

The translation model might occasionally distort numerical values when breaking down unstructured text into discrete nodes, leading to logical incoherence.

Example Context: Josh buys a house for \$80,000 and increases its value by 150%.

```
<generate>
{
  node_id: 5
  parents: 1,3
  content: The original value of the
           house is $80,000 and it has
           increased by 150%
}
</generate>
<generate>
{
  node_id: 6
  parents: 5
  content: The increase in value is
           0.15 * $80,000 = $12,000
}
</generate>
```

Resolution: Here, Node 6 mistakenly calculates the increase by multiplying by 0.15 (15%) instead of 1.5 (150%). Our **Parent-Child Coherence** verification explicitly detects this mathematical and logical disconnect between the parent (Node 5) and child (Node 6), triggering refinement to ensure zero information distortion.

Figure 9: An example of Parent-Child Incoherence.

G Training Data

G.1 Supervised Fine-Tuning (SFT) Data

- **Math Reasoning.** We use the full 40.3k samples from the DeepScaleR dataset.
- **Code Generation.** We curate 12k samples from the KodCode-V1-SFT-R1 subset, balancing task difficulty (*Easy* : *Middle* : *Hard* = 4 : 3 : 3) and task types (*Complete* : *Instruct* : *Online_Judge* = 4.5 : 4.5 : 1).

G.2 Reinforcement Learning (RL) Data

- **Math Reasoning.** We sample 5,000 problems from the DeepScaleR dataset. Problems are attempted four times by Qwen3-8B and stratified by difficulty based on success count. Problems failed in all attempts are discarded, followed by stratified sampling with a ratio of 1 : 1 : 1.5 : 1.5.
- **Code Generation.** We directly employ the KodCode-Light-RL-10K subset.

G.3 Data Leakage Check

To prevent potential data leakage, we first collected all evaluation test sets relevant to our tasks. After selecting the training data for both supervised fine-tuning and reinforcement learning, we cross-checked each training sample against these test sets. Any overlapping instances would have been removed. Our inspection confirmed that the training sets contain no examples from the test sets, ensuring a clean separation between training and evaluation data.

H Ablation Study Details

Table 4 reports the detailed ablation results of the proposed length-reducing rewards across multiple math reasoning benchmarks. We progressively remove the connectivity reward and ERS information ratio, to examine their individual contributions. The results show that removing these components leads to a substantial increase in reasoning length and a consistent degradation in accuracy, especially on more challenging benchmarks such as AIME24, highlighting their importance in controlling reasoning efficiency without sacrificing correctness.

The Prompt for Graph-Structured Translation

You are a helpful AI Assistant that provides well-reasoned and detailed responses. I will provide you with a reasoning process generated by an LLM. I need you to translate the reasoning process into a directed graph structure. Respond in the following format: `<think> \n...\n </think> \n <answer> \n...\n </answer>`

Besides, you must comply with below conditions:

1. During the `<think>` phase you should organize the chain of thought using below tags:

- known: known conditions and goals that can be found in the question.
- generate: from the current reasoning state, generate one or more new reasoning steps. It represents a step forward in the process of reasoning.
- aggregate: merge multiple steps or jointly reason over them to produce a new reasoning step.
- reflection: go back to a previous reasoning step. Used to re-examine the correctness of a step or process.
- refine: improve the current node. It is a refined modification of a certain node's statement, without producing a substantial step forward in the reasoning process.
- associative thinking: comparing the current reasoning graph structure with other similar graph structures, in order to facilitate the current reasoning process. For example, when solving a math problem, recalling the solution methods used in previous similar problems.
- reverse thinking: starting from the goal of the problem, considering possible solution paths, and filtering them with the given conditions. This builds a abstract reverse reasoning path from the goal to the conditions, from the unknown to the known. At this stage, you do not need to perform specific actions to get the answer. You just need to use reverse thinking to think about the reasoning method. The specific reasoning will be performed in the following tags.

2. At each further reasoning step you must choose one of these tags and wrap that step's output with the chosen tag. For example: `<generate>...</generate>`

3. The complete think phase must start with `<known>...</known>`, and the final inference tag must include the final result of the question.

4. The tag content inside is a series of thinking steps, organized in a node based manner with `node_id` and `parents`. You need to ensure that the thinking process is coherent and effective, and ultimately these nodes can be organized into a directed graph. The format example for each node is as follows:

```
{
  node_id: The unique identifier of a node, usually an integer, increasing from 1.
  parents: A list of parent node IDs for this node, used to establish inference dependencies. If there is no parent node, you can fill in none.
  content: The content of this step
}
```

5. For the content wrapped in different tags, there are the following formal requirements:

- known: It wraps one or more nodes, and the parents of these nodes should all be "none".
- generate: It wraps one or more nodes, (1) If it wraps one node, the parents of this nodes should be a single node. (2) If it wraps two or more nodes, the parents of these nodes should be a same single node.
- aggregate: It wraps one node, and the parent of this node should be multiple nodes.
- reflection: It wraps one node, and the parent of this node should be one or more nodes. Its `parent_ids` must include the last node of the current reasoning chain.
- refine: It wraps one node, and the parent of this node should be the last node in the current reasoning chain.
- associative thinking: It wraps one node, and the parent of this node should be one or more nodes.
- reverse thinking: It wraps one node, and the parent of this node should be one or more nodes.

6. If a tag contains multiple nodes, the parents of these nodes cannot contain other nodes in the tag.

7. If a tag contains multiple nodes, the nodes should be separated by commas. Within a node, different tags do not require commas and should be separated by line breaks.

8. Coding Format Rules (CRITICAL):

- The content inside `<answer>` must be PURE Python code. Do NOT include any XML tags (like `</think>`) inside `<answer>`.
- NO INDENTATION for top-level definitions: The `import` statements and the `def function_name(...)` line MUST start at the very beginning of the line (column 0). Do NOT add extra spaces before `def`.
- Self-Contained: Include all necessary imports (e.g., `from typing import List`).

9. Engineering Safety Rules:

- No Side Effects: DO NOT modify the input arguments in-place (e.g., use `sorted(nums)` instead of `nums.sort()`).
- Strict Signature: Use the EXACT function name and argument names provided in the prompt, even if they contain typos. Do not change the API.

Please strictly follow the above format and requirements.

Below I'll give you several examples:

Figure 10: The prompt template for "Graph-Structured Translation". Criteria 8 and 9 are specifically added to accommodate code generation task.

The First Example of Mathematical Reasoning

Question:

Find the smallest number (n) such that given any (n) distinct numbers from ($\{1, 2, 3, \dots, 999\}$), one can choose four different numbers (a, b, c, d) such that ($a + 2b + 3c = d$).

Response:

```

<think>
  <known>
    {
      node_id:1
      parents:none
      content:The set is {1, 2, ..., 999}
    },
    {
      node_id:2
      parents:none
      content:We are to find the smallest n such
      that any subset of size n contains distinct
      a,b,c,d with  $a + 2b + 3c = d$ 
    },
    {
      node_id:3
      parents:none
      content:a, b, c, d must be distinct elements
      from the chosen n numbers
    }
  </known>
  <reverse thinking>
    {
      node_id:4
      parents:2
      content:To find the minimal n that forces
      the equation  $a + 2b + 3c = d$  to hold among
      any n-element subset, we consider the
      maximal size of a subset that avoids this
      equation; then n is one more than that
      maximum.
    }
  </reverse thinking>

```

Continuation in the second column.

Continued from the previous column.

```

<generate>
  {
    node_id:5
    parents:4
    content:Let  $S \subseteq \{1, \dots, 999\}$  be a subset
    with no four distinct elements satisfying
     $a + 2b + 3c = d$ . We want to maximize  $|S|$ .
  }
</generate>
<associative thinking>
  {
    node_id:6
    parents:5
    content:This resembles extremal
    combinatorics problems like Schur's
    theorem or sum-free sets. In such
    problems, choosing large numbers often
    avoids linear equations because the RHS
    (d) would exceed the universe bound.
  }
</associative thinking>
<generate>
  {
    node_id:7
    parents:6
    content:If we choose only large numbers,
    say all numbers  $> T$ , then  $a + 2b + 3c \geq$ 
 $1*T + 2*T + 3*T = 6T$ , so if  $6T > 999$ ,
    then  $d = a + 2b + 3c \notin \{1, \dots, 999\}$ , hence
    cannot be in S.
  }
</generate>
.....

```

I omitted the subsequent reasoning and answer generation process. In this example, you mainly learned how to use reverse thinking and associative thinking.

Figure 11: A mathematical example illustrating reverse and associative thinking.

The Second Example of Mathematical Reasoning

Question:

Find the sum of all integer bases $b > 9$ for which $17_{\{b\}}$ is a divisor of $97_{\{b\}}$

Response:

```

<think>
  <known>
    {
      node_id:1
      parents:none
      content:b>9
    },
    {
      node_id:2
      parents:none
      content:17_{b} is a divisor of 97_{b}
    },
    {
      node_id:3
      parents:none
      content:b is an integer
    }
  </known>
  <generate>
    {
      node_id:4
      parents:2
      content:17_{b}=b+7
    },
    {
      node_id:5
      parents:2
      content:97_{b}=9*b+7
    },
  </generate>
  <aggregate>
    {
      node_id:6
      parents:2,4,5
      content: 9*b+7=k(b+7),k>0,k is an integer
    },
  </aggregate>
  <generate>
    {
      node_id:7
      parents:6
      content:b=(7-7k)/(k-9),1<k<9,k is an integer
    }
  </generate>
  <associative thinking>
    {
      node_id:8
      parents:7
      content:When dealing with this type of problem
      before, I used the enumeration method, and I can
      apply the same method here as well.
    }
  
```

Continuation in the second column.

Continued from the previous column.

```

    }
  </associative thinking>
  <aggregate>
    {
      node_id:9
      parents:1,3,7,8
      content:Next, I should enumerate k under
      the condition that 1<k<9, k is an integer,
      and calculate when b satisfies the
      condition that b>9 and b is an integer.
    },
  </aggregate>
  <generate>
    {
      node_id:10
      parents:9
      content:1.if k=2,b=1,false.
    },
  </generate>
  <reflection>
    {
      node_id:16
      parents:9
      content:7.if k=8,b=49,true.
    }
  </reflection>
  <aggregate>
    {
      node_id:17
      parents:6,16
      content:But wait: Also b+7=? and 9*b+7=?
      Possibly b+7=56 and 9*b+7=448?
      448/56=8 Yes.
    }
  </aggregate>
  <reflection>
    {
      node_id:18
      parents:10,11,12,13,14,15,16
      content:Sum=21+49=70
    }
  </reflection>
  </aggregate>
</think>
<answer>
  70
</answer>
  
```

Intermediate enumeration steps are omitted; see the schematic diagram for details.

Figure 12: A mathematical example in graph-structured format.

Question:

Find the sum of all integer bases $b > 9$ for which 17_b is a divisor of 97_b

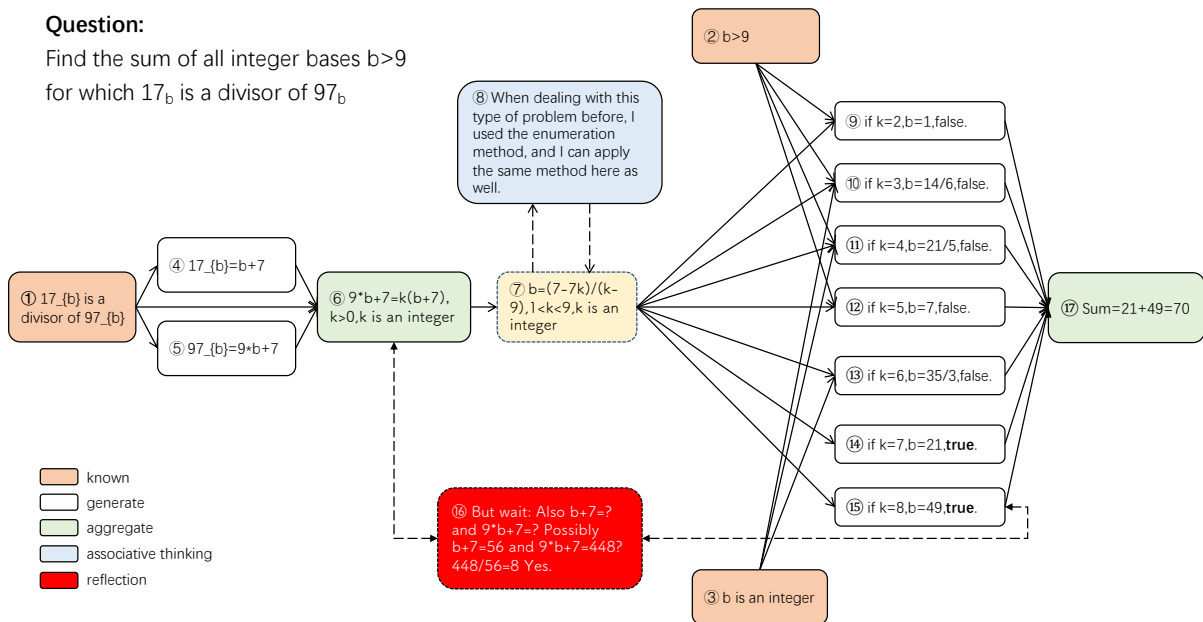


Figure 13: Graph visualization of the mathematical example.

Question:

```
def coin_change(coins: List[int], amount: int) -> int:
    """ You are given an integer array coins representing coins of different denominations
    and an integer amount representing a total amount of money. Return the fewest number
    of coins that you need to make up that amount. If that amount of money cannot be made
    up by any combination of the coins, return -1.
    >>> coin_change([1, 2, 5], 11) == 3
    >>> coin_change([2], 3) == -1
    >>> coin_change([1], 0) == 0
    >>> coin_change([1], 1) == 1
    >>> coin_change([1], 2) == 2
    """
```

Response:

```
<think>
<known>
{
  node_id: 1,
  parents: none,
  content: "Input: 'coins' (List[int]), 'amount' (int). Goal: Find minimum
  number of coins to sum to 'amount'."
},
{
  node_id: 2,
  parents: none,
  content: "Constraints: Return -1 if impossible. 'amount' >= 0 (implied
  by context/doctests). Coins are positive integers."
},
{
  node_id: 3,
  parents: none,
  content: "Edge Cases: amount is 0 (expect 0). coins list is empty
  (expect -1 unless amount is 0). amount cannot be formed."
}
</known>
<associative thinking>
{
  node_id: 4,
  parents: 1,
  content: "Pattern Recognition: This is the classic 'Coin Change'
  problem, which is a variation of the Unbounded Knapsack Problem.
  Potential approaches: BFS (shortest path in a graph) or Dynamic
  Programming (optimal substructure)."
}
</associative thinking>
<reflection>
{
  node_id: 5,
  parents: 4,
  content: "Check Greedy Approach: If we always take the largest coin
  <= amount, does it work? \nCounter-example: coins=[1, 3, 4],
  amount=6. \nGreedy: 4 + 1 + 1 = 3 coins. \nOptimal: 3 + 3 = 2 coins.
  \nConclusion: Greedy is not optimal. Must use DP or BFS."
}
</reflection>
<reverse thinking>
{
  node_id: 6,
  parents: 5,
  content: "Logic Derivation: To find dp[x] (min coins for amount x),
  the last coin added must be some coin 'c'. Thus, dp[x] = min(dp[x-c]
  + 1 for all c in coins. We solve from 0 up to amount."
}
</reverse thinking>
<generate>
{
  node_id: 7,
  parents: 6,
  content: "Plan (DP Bottom-Up):\n1. Initialize array 'dp' of size
  amount + 1.\n2. Fill 'dp' with a value > amount (representing infinity).
  dp[0] = 0.\n3. Iterate i from 1 to amount.\n4. For each coin, if i >=
  coin, dp[i] = min(dp[i], dp[i-coin] + 1).\n5. Final result is dp[amount]
  (or -1 if it's still infinity)."
}
</generate>
<refine>
{
  node_id: 8,
  parents: 7,
```

Continuation in the second column.

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```
content: "Optimization: We can check 'coin <= i' inside the inner loop to
avoid negative index access. The 'infinity' value can be 'amount + 1' since
the worst case is all 1s (count = amount)."
```

```
</refine>
<generate>
{
  node_id: 9,
  parents: 8,
  content: "Implementation Draft:\nndef coin_change(coins, amount):\n
  dp = [amount + 1] * (amount + 1)\n  dp[0] = 0\n  for i in range(1, amount
  + 1):\n    for c in coins:\n      if i >= c:\n        dp[i] = min(dp[i], dp[i-c] + 1)\n
  return dp[amount] if dp[amount] <= amount else -1"
```

```
</generate>
<reflection>
{
  node_id: 10,
  parents: 9,
  content: "Dry Run Trace:\nInput: coins=[1, 2], amount=3.\nInit: dp=[0,
  4, 4, 4] (inf=4)\ni=1: c=1 -> dp[1]=min(4, dp[0]+1)=1. dp=[0, 1, 4,
  4]\ni=2: c=1 -> dp[2]=min(4, dp[1]+1)=2. c=2 -> dp[2]=min(2,
  dp[0]+1)=1. dp=[0, 1, 1, 4]\ni=3: c=1 -> dp[3]=min(4, dp[2]+1)=2. c=2
  -> dp[3]=min(2, dp[1]+1)=2. dp=[0, 1, 1, 2]\nResult: 2. Correct."
```

```
</reflection>
<aggregate>
{
  node_id: 11,
  parents: 2, 3, 10,
  content: "Final Verification: Handles edge case amount=0 correctly
  (dp[0]=0). Handles impossible case correctly (returns -1). Code structure
  is compliant with constraints."
}
</aggregate>
</think>
<answer>
from typing import List

def coin_change(coins: List[int], amount: int) -> int:
    """ You are given an integer array coins representing coins of different
    denominations and an integer amount representing a total amount of
    money. Return the fewest number of coins that you need to make up that
    amount. If that amount of money cannot be made up by any combination
    of the coins, return -1.
    >>> coin_change([1, 2, 5], 11) == 3
    >>> coin_change([2], 3) == -1
    >>> coin_change([1], 0) == 0
    >>> coin_change([1], 1) == 1
    >>> coin_change([1], 2) == 2
    """
    if amount < 0:
        return -1
    if amount == 0:
        return 0
    max_val = amount + 1
    dp = [max_val] * (amount + 1)
    dp[0] = 0
    for i in range(1, amount + 1):
        for coin in coins:
            if i >= coin:
                dp[i] = min(dp[i], dp[i - coin] + 1)
    return dp[amount] if dp[amount] <= amount else -1
</answer>
```

Figure 14: A code completion example.

Question:

You are given an integer array `coins` representing coins of different denominations and an integer `amount` representing a total amount of money. Return the fewest number of coins that you need to make up that amount. If that amount of money cannot be made up by any combination of the coins, return -1.

Function Signature: `def coin_change(coins: List[int], amount: int) -> int:`

Response:

```
<think>
<known>
{
  node_id:1,
  parents:none,
  content:Input: List of integers 'coins', Integer 'amount'. Goal:
  Find Minimum count of coins to sum to 'amount'.
},
{
  node_id:2,
  parents:none,
  content:Constraints: 'amount' can be 0 (return 0). If
  unsolvable, return -1. Coins are positive integers.
},
{
  node_id:3,
  parents:none,
  content:Edge Cases: Coins list is empty; Amount is negative;
  Large amount vs small coins (performance).
}
</known>
<reverse thinking>
{
  node_id:4,
  parents:1,
  content:To reach the target 'amount' with minimum coins, the
  last coin added must be some coin 'c' from the list. This
  means the problem reduces to finding min coins for 'amount
  - c'. This recursive structure implies we solve from 0 up to
  'amount'.
}
</reverse thinking>
<associative thinking>
{
  node_id:5,
  parents:4,
  content:This structure looks like the 'Unbounded Knapsack
  Problem'. A common approach is Greedy (always take largest)
  or Dynamic Programming. Let's try to verify if Greedy
  works because it is faster.
}
</associative thinking>
<reflection>
{
  node_id:6,
  parents:5,
  content:Verifying Greedy: Counter-example check. Coins=[1,
  3, 4], Amount=6. Greedy path: Take 4, remain 2. Take 1,
  remain 1. Take 1. Total 3 coins (4+1+1). Optimal path: Take 3,
  remain 3. Take 3. Total 2 coins (3+3). So Greedy FAILS.
  Must use Dynamic Programming.
}
</reflection>
<generate>
{
  node_id:7,
  parents:6,
  content:Plan (DP approach):
  1. Create array dp[amount + 1] initialized to 'infinity'
  (amount + 1).
  2. Set dp[0] = 0.
  3. Iterate i from 1 to amount.

```

Continuation in the second column.

Continued from the previous column.

```

  4. For each coin, if i >= coin, dp[i] = min(dp[i], dp[i-coin] + 1).
}
</generate>
<aggregate>
{
  node_id:8,
  parents:2,3,7,
  content:Combine DP Plan with Edge Cases:
  If amount is 0, return 0 immediately.
  After DP loop, if dp[amount] > amount, implies no solution,
  return -1.
}
</aggregate>
<generate>
{
  node_id:9,
  parents:8,
  content:Implementation Code:
  def coin_change(coins, amount):
  if amount == 0: return 0
  max_val = amount + 1
  dp = [max_val] * (max_val)
  dp[0] = 0
  for i in range(1, amount + 1):
  for coin in coins:
  if i - coin >= 0:
  dp[i] = min(dp[i], dp[i - coin] + 1)
  return dp[amount] if dp[amount] != max_val else -1
}
</generate>
<reflection>
{
  node_id:10,
  parents:9,
  content:Dry Run Trace: Input: coins=[1, 2], amount=3. Init: dp=[0,
  4, 4, 4]. i=1: coin=1 -> dp[1]=min(4, dp[0]+1)=1. dp=[0,1,4,4].
  i=2: coin=1 -> dp[2]=min(4, dp[1]+1)=2. coin=2 -> dp[2]=min(2,
  dp[0]+1)=1. dp=[0,1,1,4]. i=3: coin=1 -> dp[3]=min(4,
  dp[2]+1)=2. coin=2 -> dp[3]=min(2, dp[1]+1)=2. dp=[0,1,1,2].
  Result: 2. Logic holds.
}
</reflection>
</think>
<answer>
from typing import List
def coin_change(coins: List[int], amount: int) -> int:
    """
    Computes the fewest number of coins needed to make up the
    amount.
    Returns -1 if impossible.
    """
    if amount == 0:
        return 0
    # Initialize DP array. amount + 1 acts as infinity.
    max_val = amount + 1
    dp = [max_val] * (amount + 1)
    dp[0] = 0
    for i in range(1, amount + 1):
        for coin in coins:
            if coin <= i:
                dp[i] = min(dp[i], dp[i - coin] + 1)
    return dp[amount] if dp[amount] <= amount else -1
</answer>

```

Figure 15: An instruction-following code generation example.

Question:

You are given an integer amount representing a total amount of money and a list of coins of different denominations. Write a program to calculate the fewest number of coins that you need to make up that amount. If that amount of money cannot be made up by any combination of the coins, print -1.

Input Format:

- The first line contains an integer 'amount'.
- The second line contains a sequence of integers 'coins' separated by spaces.

Output Format:

- Print a single integer representing the minimum number of coins (or -1).

Example 1:Input:11 1 2 5 Output:3

Example 2:Input:3 2 Output:-1

Format: Generate an executable Python function to solve the given problem. The function should read input from 'stdin' and write the output to 'stdout'. Do not directly test on the sample inputs. Ensure that when the Python program runs, it reads the inputs, executes the algorithm, and writes the output to 'stdout'.

Response:

```
<think>
<known>
{
  node_id: 1,
  parents: none,
  content: "Task: Solve Coin Change problem (Min coins for 'amount').\nInput Source: stdin (Standard Input).\nOutput Target: stdout (Standard Output).\nInput Format: Line 1 = amount (int), Line 2 = coins (space-separated ints).",
},
{
  node_id: 2,
  parents: none,
  content: "Constraints: Print -1 if impossible. Standard integer limits. Time complexity matters (Standard OJ limits usually ~1-2s).",
}
</known>
<associative thinking>
{
  node_id: 3,
  parents: 1,
  content: "Algorithm Selection: This is the 'Coin Change' problem (Unbounded Knapsack). DP (Dynamic Programming) is the standard optimal solution. BFS is also possible but DP is easier to implement for just counting."
}
</associative thinking>
<reverse thinking>
{
  node_id: 4,
  parents: 1,
  content: "IO Strategy: Unlike function completion, we must parse raw strings.\nReading: `sys.stdin.read().split()` is robust for handling whitespace/newlines.\nParsing sequence: First token is 'amount', subsequent tokens are 'coins'."
}
</reverse thinking>
<generate>
{
  node_id: 5,
  parents: 3, 4,
  content: "Plan:\n1. Import sys.\n2. Read all stdin content.\n3. Parse 'amount' and 'coins' list.\n4. Initialize DP array 'dp' size [amount + 1] with 'amount + 1' (infinity).\n5. Set dp[0] = 0.\n6. Iterate i from 1 to amount: for each coin, update dp[i].\n7. Print result."
}
</generate>
<refine>
{
  node_id: 6,
  parents: 5,
  content: "Robustness Check: What if the coins line is empty? (Though problem implies valid coins). What if amount is 0? The DP initialization covers amount=0 correctly. We should ensure we strip inputs properly."
}
</refine>
```

Continued from the previous column.

```
<generate>
{
  node_id: 7,
  parents: 6,
  content: "Implementation Logic:\n\nimport sys\n\ndef solve():\n  input_data = sys.stdin.read().split()\n  if not input_data: return\n  amount = int(input_data[0])\n  coins = [int(x) for x in input_data[1:]]\n  # DP Logic\n  dp = [amount + 1] * (amount + 1)\n  dp[0] = 0\n  for i in range(1, amount + 1):\n    for c in coins:\n      if i >= c:\n        dp[i] = min(dp[i], dp[i-c] + 1)\n  result = dp[amount] if dp[amount] <= amount else -1\n  print(result)\n\nif __name__ == '__main__':\n  solve()
}
</generate>
<reflection>
{
  node_id: 8,
  parents: 7,
  content: "Dry Run (Mental Sandbox):\nInput:\n11\n1 2 5\nTokens: ['11', '1', '2', '5']\namount = 11\ncoins = [1, 2, 5]\nDP Init: dp size 12.\ni=1..11 loop runs.\nFinal dp[11] will be 3 (5+5+1 or similar combo).\nOutput: 3.\nInput:\n3\n2\nTokens: ['3', '2']\namount = 3, coins = [2]\ndp[1] -> inf\ndp[2] -> 1\ndp[3] -> inf (since 3-2=1, dp[1] is inf)\nOutput: -1.\nLogic holds."
}
</reflection>
<aggregate>
{
  node_id: 9,
  parents: 7, 8,
  content: "Final Polish: Combine logic into the required executable format. Ensure strict 'sys.stdin' usage and no extra prompt text (like 'Enter number:') as it breaks OJ graders."
}
</aggregate>
</think>
<answer>
import sys
def solve():
  input_data = sys.stdin.read().split()
  if not input_data:
    return
  amount = int(input_data[0])
  coins = [int(x) for x in input_data[1:]]
  # dp[i] = min coins to make i
  max_val = amount + 1
  dp = [max_val] * (amount + 1)
  dp[0] = 0
  for i in range(1, amount + 1):
    for coin in coins:
      if coin <= i:
        dp[i] = min(dp[i], dp[i - coin] + 1)
  print(dp[amount] if dp[amount] <= amount else -1)

if __name__ == "__main__":
  solve()
</answer>
```

Continuation in the second column.

Figure 16: A competitive programming code generation example.

The Prompt for Graphical CoT Verification and Refinement

You are a strict evaluator for graph-structured reasoning.

Each reasoning node is associated with:

- a label indicating its thinking mode,
- textual content,
- and optional parent node(s).

Your task is to evaluate the reasoning graph from two dimensions.

Dimension 1: Label–Content Consistency

For each node, judge whether its content complies with the definition of its assigned label.

The label definitions are as follows:

- known:** Known conditions explicitly stated in the question. This node should not depend on any parent.
- generate:** Generate one or more new reasoning steps from the current reasoning state.
- aggregate:** Combine multiple reasoning steps to produce a new conclusion. This node should have multiple parents.
- feedback / reflection:** Go back to a previous reasoning step to re-examine its correctness.
- refine:** Improve or rephrase an existing reasoning step without introducing substantial new progress.
- associative thinking:** Draw analogies to similar problems or reasoning graph structures to facilitate reasoning.
- reverse thinking:** Start from the target goal and reason backward, filtering possible solution paths using given conditions.

Score each node with a value between 0.0 and 1.0, where:

- 1.0 indicates perfect consistency with the label definition,
- 0.0 indicates complete mismatch or contradiction.

Dimension 2: Parent–Child Semantic Coherence

For nodes with parent(s), judge whether the child node logically follows from and builds upon its parent node(s).

Evaluation criteria include:

- The child should not contradict the parent reasoning.
- Repeating or refining parent reasoning in a consistent way should receive a high score.
- Introducing a valid new reasoning step that logically follows should receive a high score.
- Contradictions or violations of parent logic should receive a low score.

If a node has no parent (e.g., a known node), it is considered coherent by default.

Score semantic coherence with a value between 0.0 and 1.0.

Output Requirement

For each evaluation, respond only with a numeric score between 0.0 and 1.0.

Figure 17: Prompt for “Graphical CoT Verification and Refinement”.