CSULoRA: Closest Safe Update Low-Rank Adaptation

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Abstract

Low-rank adaptation has become a standard method for parameter-efficient fine-tuning of large language models, but even small amounts of unsafe or adversarial fine-tuning data can substantially weaken the safety behavior of aligned models. Existing safety-preserving LoRA methods often rely on hard interventions such as projection, pruning, thresholding, or additional training objectives. While these methods can suppress unsafe update directions, they may also remove task-relevant information or require extra tuning. We introduce CSULoRA, a post-hoc method for correcting trained LoRA adapters through closest safe update estimation. CSULoRA estimates a safety-aligned subspace from the weight displacement between a safety-aligned model and its corresponding base checkpoint. It then decomposes each LoRA update into fully aligned, partially aligned, and off-subspace components. Instead of discarding components outside the estimated safety subspace, CSULoRA solves a closed-form penalized minimum-change problem that preserves the fully aligned component while smoothly attenuating potentially unsafe directions according to their relative energy. In adversarial fine-tuning experiments, CSULoRA substantially reduces attack success rate while preserving most of the utility gains obtained from standard LoRA fine-tuning[1].

1 Introduction

Low-rank adaptation (LoRA) is widely used for parameter-efficient fine-tuning because it adapts large language models through small trainable low-rank updates while keeping the base model frozen (Hu et al., 2022). However, recent work shows that fine-tuning aligned models can weaken safety behavior and increase compliance with harmful prompts, even when the fine-tuning data is small or not explicitly intended to be adversarial (Yang et al., 2023; Qi et al., 2024; Bianchi et al., 2024). This motivates post-hoc methods that modify trained LoRA adapters to reduce safety degradation without retraining the full model.

Existing safety-preserving LoRA and post-hoc safety restoration methods include projection-based correction (Hsu et al., 2024), pruning-based correction (Ao et al., 2025), safety-module-based adaptation (Li et al., 2025), Fisher- or geometry-guided regularization (Das et al., 2025), task-arithmetic restoration (Bhardwaj et al., 2024), and layer-wise merging (Djuhera et al., 2025) or delta correction (Lu et al., 2025) approaches. While effective in some settings, these methods may discard task-relevant information or require additional training, pruning decisions, thresholding, regularization objectives, or external safety-vector construction.

We introduce CSULoRA, a post-hoc closest safe update method for LoRA adapters. CSULoRA decomposes each trained adapter update into components that differ in their overlap with an estimated safety-aligned subspace. Instead of removing the off-subspace component, it solves a penalized minimum-change optimization problem that preserves the fully aligned component exactly while smoothly attenuating less alignment-consistent directions. CSULoRA requires no retraining, no additional trainable parameters, and no validation-based threshold selection. Given a trained LoRA adapter and a base/aligned model pair, it deterministically computes a corrected adapter in closed form.

Our experiments study adversarial fine-tuning, where benign constrained instruction-following data is mixed with a small fraction of unsafe examples. Compared with standard LoRA and safety-preserving baselines, CSULoRA substantially lowers attack success rate while preserving most of the utility improvement from LoRA fine-tuning and

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[1] https://github.com/Oleksandr-MB/NLPAICS2026_CSULoRA

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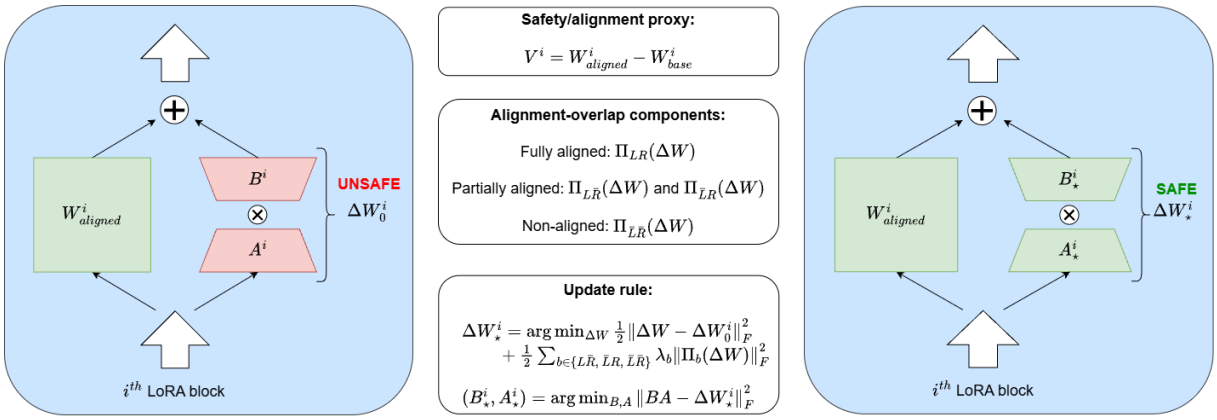

Figure 1: Overview of CSULoRA. Given an aligned model and its corresponding base checkpoint, CSULoRA first estimates an alignment displacement $V = W_{\text{aligned}} - W_{\text{base}}$. For each trained LoRA update $\Delta W = BA$, it constructs left and right alignment-aware projectors from V, decomposes ΔW into fully aligned, partially aligned, and non-aligned components, and solves an optimization problem that preserves the fully aligned component while softly penalizing energy in the remaining blocks. The corrected update $\Delta W^\star$ is then refactored back into LoRA matrices $B^\star A^\star$, preserving the original adapter structure.

maintaining general capability.

2 Related Work

Parameter-efficient adaptation of language models. Parameter-efficient fine-tuning has become a standard strategy for adapting large language models without updating all model parameters. Early adapter-based methods insert small trainable modules into frozen pretrained networks, reducing task-specific parameter cost while preserving most of the benefits of full fine-tuning (Houlsby et al., 2019). LoRA further simplifies this paradigm by injecting low-rank trainable matrices into existing weight layers, keeping the backbone frozen and adding no inference-time architectural depth (Hu et al., 2022). Because LoRA offers a favorable trade-off between adaptation quality and computational cost, it has become a widely used mechanism for customizing instruction-tuned LLMs. However, the same flexibility also creates a safety risk: small low-rank updates can substantially alter model behavior, including the refusal and safety patterns learned during alignment.

Safety degradation under fine-tuning. Recent work has shown that the safety behavior of aligned LLMs is fragile under downstream fine-tuning. Shadow Alignment demonstrates that a small number of malicious examples can subvert safety-aligned models while largely preserving their general helpfulness (Yang et al., 2023). Similarly, fine-tuning aligned language models on adversarial or even benign user datasets can weaken safety guardrails and increase harmful compliance (Qi et al., 2024). These findings motivate methods that protect alignment during or after adaptation, especially in settings where users fine-tune open or API-accessible models. Our work follows this line of research but focuses specifically on post-hoc correction of trained LoRA adapters, where the objective is to recover safety without retraining the full model or discarding the utility gained through adaptation.

Safety-preserving LoRA adaptation. Several recent methods modify LoRA training or post-processing to reduce safety degradation. SafeLoRA projects selected LoRA weights onto a safety-aligned subspace estimated from the difference between base and aligned checkpoints (Hsu et al., 2024). This makes it closely related to CSULoRA, since both use the base–aligned displacement as a proxy for alignment-relevant directions. However, SafeLoRA applies a hard projection, whereas CSULoRA decomposes each update into alignment-overlap blocks and applies a closed-form soft attenuation rule. SPLoRA instead identifies and prunes LoRA layers that are likely to harm safety alignment using a distance-guided criterion (Ao et al., 2025). SaLoRA introduces a fixed safety module and task-specific initialization to preserve safety during adaptation (Li et al., 2025). AlignGuard-LoRA formulates safety-preserving fine-tuning as a regularized adaptation problem that constrains alignment drift during training (Das et al., 2025). These methods show that LoRA updates contain

safety-relevant structure, but they often rely on hard projection, pruning, additional safety modules, regularization losses, or method-specific hyperparameters. In contrast, CSULoRA is a deterministic post-hoc adapter surgery method: it requires no additional training and preserves the original LoRA rank and adapter structure.

Post-hoc safety restoration and update-space correction. A complementary line of work restores safety after fine-tuning by modifying model deltas or merging model weights. RESTA uses task arithmetic to add a safety vector back into a compromised model (Bhardwaj et al., 2024). SafeDelta estimates safety degradation in the fine-tuning delta and applies safety-aware post-training correction to preserve utility while limiting safety loss (Lu et al., 2025). SafeMERGE selectively merges layers from fine-tuned and safety-aligned models based on layer-wise deviation from safe behavior (Djuhera et al., 2025). These methods demonstrate the effectiveness of post-hoc correction, but they typically operate at the model-delta or layer-merging level. CSULoRA differs by operating directly on trained LoRA adapter matrices and by deriving a minimum-change closed-form update that attenuates less alignment-consistent components rather than replacing or merging full layers.

Projection and subspace methods for preserving model behavior. CSULoRA is also related to geometric approaches that constrain update directions in parameter space. OPLoRA uses double-sided orthogonal projections to prevent catastrophic forgetting by restricting LoRA updates away from dominant singular directions of the frozen weights (Xiong and Xie, 2025). CSULoRA adopts a related double-sided projection perspective, but with a different goal and subspace definition: instead of protecting pretrained knowledge from forgetting, it estimates alignment-relevant input and output subspaces from the base–aligned displacement and softly penalizes LoRA components outside these subspaces. This makes CSULoRA a safety-oriented extension of projection-based adapter correction, positioned between hard projection methods and broader post-hoc safety restoration techniques.

3 Proposed Approach

We propose CSULoRA, a post-hoc modification method for trained LoRA adapters that performs safety-oriented adapter surgery without additional training. Given a LoRA adapter trained on top of a safety-aligned model, CSULoRA rewrites each layer-wise update by solving a minimum-change optimization problem. In contrast to hard projection methods, which may discard the entire off-subspace component of an update, CSULoRA preserves the component lying fully inside an estimated safety-aligned subspace and softly attenuates the remaining components according to their relative energy (Figure 1).

Let the original LoRA update for layer i be

$$\Delta W_0^i = B^i A^i,$$

where $A^i \in \mathbb{R}^{r \times d_{\text{in}}}$ and $B^i \in \mathbb{R}^{d_{\text{out}} \times r}$. To estimate safety-aligned directions for this layer, we use the weight displacement between a safety-aligned checkpoint and its corresponding pre-alignment base checkpoint:

$$V^i = W^i_{\text{aligned}} - W^i_{\text{base}}.$$

Here, W_{aligned} denotes the safety-aligned instruction-tuned checkpoint (Rafailov et al., 2023; Ouyang et al., 2022), while W_{base} denotes the corresponding pre-alignment base model checkpoint. The LoRA adapter itself is applied on top of W_{aligned}; the base checkpoint is used only to estimate the alignment-induced displacement V^i. Following projection-based safety-preserving LoRA methods (Hsu et al., 2024; Ao et al., 2025), we treat the dominant subspaces of V^i as practical proxies for alignment-relevant directions.

Because a weight matrix maps from an input space to an output space, we estimate alignment-relevant subspaces on both sides of the update. This follows the double-sided projection perspective of OPLoRA, where left and right projectors are used to control how LoRA updates interact with the singular structure of a weight matrix (Xiong and Xie, 2025). In CSULoRA, however, the projectors are constructed from the alignment delta V^i. The dominant column space of V^i defines an output-space basis U^i_L, while the dominant row space of V^i defines an input-space basis U^i_R. These bases induce the orthogonal projectors

$$P^i_L = U^i_L (U^i_L)^\top, \qquad P^i_R = U^i_R (U^i_R)^\top.$$

To reduce computational cost, we approximate these dominant subspaces using a randomized low-rank range finder with power iterations (Halko

et al., 2011; Tropp and Webber, 2023). The effective rank is chosen adaptively by retaining enough singular-value energy to explain 95% of the alignment-displacement energy.

Given the projectors P_L^i and P_R^i, we decompose the original LoRA update ΔW_0^i into four projection blocks:

$$\begin{aligned}\Delta W_{LR}^i &= P_L^i \Delta W_0^i P_R^i,\\ \Delta W_{L\bar{R}}^i &= P_L^i \Delta W_0^i - \Delta W_{LR}^i,\\ \Delta W_{\bar{L}R}^i &= \Delta W_0^i P_R^i - \Delta W_{LR}^i,\\ \Delta W_{\bar{L}\bar{R}}^i &= \Delta W_0^i - P_L^i \Delta W_0^i - \Delta W_0^i P_R^i + \Delta W_{LR}^i.\end{aligned}$$

The block ΔW_{LR}^i lies in both the aligned input and output subspaces, and is therefore treated as the most alignment-consistent component. The mixed blocks $\Delta W_{L\bar{R}}^i$ and $\Delta W_{\bar{L}R}^i$ lie in only one of the two aligned subspaces, while $\Delta W_{\bar{L}\bar{R}}^i$ lies outside both.

For clarity define a set of all block names:

$$\mathcal{B} = \{LR,\ L\bar{R},\ \bar{L}R,\ \bar{L}\bar{R}\}$$

The core of CSULoRA is a penalized optimization problem. We look for an optimal update $\Delta W_\star^i$ that remains close to the original LoRA update while penalizing energy in the partially aligned and non-aligned blocks:

$$\begin{aligned}\Delta W_\star^i = \arg\min_{\Delta W} \frac{1}{2} \left\|\Delta W - \Delta W_0^i\right\|_F^2 \\ + \frac{1}{2} \sum_{b \in \mathcal{B}\setminus\{LR\}} \lambda_b \left\|\Pi_b(\Delta W)\right\|_F^2,\end{aligned}$$

where $\Pi_b(\cdot)$ denotes the projection onto block b. The fully aligned block is left unpenalized. Since the four blocks are induced by orthogonal projectors, the objective decomposes blockwise and admits the closed-form solution (See the derivation in the Appendix A)

$$\Delta W_\star^i = \Delta W_{LR}^i + \sum_{b \in \mathcal{B}\setminus\{LR\}} \gamma_b \Delta W_b^i$$

with $\gamma_b = (1+\lambda_b)^{-1}$.

To avoid relying on tunable hyperparameters, we compute the penalty terms adaptively as the ratio of relative Frobenius energies of the projection blocks: i.e., for each block define

$$E_b = \|\Delta W_b^i\|_F^2,$$

We define the "reference" energy

$$E_{\text{ref}} = E_{LR} + \beta \sum_{b \in \mathcal{B}} E_b,$$

where $\beta > 0$ is a small numerical constant (in our case we use $\beta = 0.05$), that prevents overly aggressive shrinkage when the fully aligned component is small by smoothing it. Then for each non-fully-aligned block $b \in \mathcal{B} \setminus \{LR\}$, we compute the penalty term λ_b as:

$$\lambda_b = \frac{E_b}{E_{\text{ref}}}.$$

The reference energy E_{ref} uses the fully aligned block energy E_{LR} as the natural scale for alignment-consistent update magnitude. The smoothing term $\beta \sum_{b \in \mathcal{B}} E_b$ prevents the denominator from becoming too small when E_{LR} is close to zero. This choice makes the penalties scale-invariant: if all LoRA update blocks are multiplied by a constant, then both E_b and E_{ref} scale quadratically, leaving $\lambda_b = E_b/E_{\text{ref}}$ unchanged. Consequently, CSULoRA attenuates non-fully-aligned components according to their relative energy within a layer, rather than according to the absolute magnitude of the layer update. Blocks whose energy is small relative to the aligned component are mostly preserved, while blocks that dominate the update receive stronger shrinkage.

Finally, the corrected update must be written in the same rank-r LoRA form as the original adapter. We therefore approximate $\Delta W_\star^i$ by a product BA with the original LoRA dimensions:

$$\begin{aligned}(B_\star^i, A_\star^i) = \arg\min_{B,A} \left\|BA - \Delta W_\star^i\right\|_F^2, \text{ with} \\ B \in \mathbb{R}^{d_{\text{out}} \times r}, \quad A \in \mathbb{R}^{r \times d_{\text{in}}}.\end{aligned}$$

By the Eckart-Young theorem, the closest rank-r approximation to $\Delta W_\star^i$ in Frobenius norm is obtained by the truncated SVD (Eckart and Young, 1936):

$$\Delta W_\star^i \approx U_r \Sigma_r V_r^\top.$$

The reconstruction error of the SVD approximation is then the sum of singular values σ_j of $\Delta W_\star^i$:

$$\left\|\Delta W_\star^i - U_r \Sigma_r V_r^\top\right\|_F^2 = \sum_{j>r} \sigma_j^2,$$

which is exactly the cost of preserving the original LoRA rank.

We then split the rank-r approximation into LoRA factors:

$$B_\star^i = U_r \Sigma_r^{1/2}, \qquad A_\star^i = \Sigma_r^{1/2} V_r^\top.$$

The original LoRA matrices A^i and B^i are replaced by $A_\star^i$ and $B_\star^i$, preserving the adapter rank and structure.

4 Experimental Setup

Utility training and scoring. Prior safety-preserving LoRA work often evaluates utility on general instruction-following or summarization tasks such as Alpaca-style response generation and DialogSum (Hsu et al., 2024; Ao et al., 2025). In preliminary experiments, these benchmarks were not sufficiently sensitive to the type of adaptation studied here: base and fine-tuned models obtained similar BERTScore values (Zhang et al., 2020), making it difficult to objectively assess the utility improvement. We therefore evaluate utility with IFEval (Zhou et al., 2023), which directly measures constrained instruction following.

We fine-tune the models on constrained instruction-following data derived from the IF_multi_constraints_upto5 dataset. To construct this dataset, we prompt Gemma-4-31B-it to generate responses, score them with the official IFBench grader (Pyatkin et al., 2025), and retain only high-quality examples with strict prompt accuracy ≥ 0.8 and loose prompt accuracy $= 1.0$[2].

Adversarial fine-tuning setup. Although benign fine-tuning alone can degrade safety (Qi et al., 2024), we found this effect inconsistent across datasets and hyperparameters. We therefore use a controlled adversarial setting by injecting unsafe prompt-response pairs from BeaverTails (Ji et al., 2023) into the supervised fine-tuning data. The final training mixture contains 20,000 examples: 19,000 benign constrained instruction-following samples and 1,000 unsafe samples, corresponding to a 95:5 mixture ratio.

Evaluation. We evaluate utility, safety, and general capability. Utility is measured with IFEval (Zhou et al., 2023). Safety is measured on AdvBench (Zou et al., 2023). We estimate attack success rate (ASR) using a regex-based refusal detector covering common refusal patterns (See Appendix B). General capability is measured with ARC-Challenge (Clark et al., 2018), used as a proxy benchmark for general knowledge preservation.

Hyperparameters. For both studied models, we train for 3 epochs with maximum sequence length 2048, batch size 1, gradient accumulation over 16 steps, learning rate 5×10^{-5}, weight decay 0.01, maximum gradient norm 1.0, and random seed 42. Gradient checkpointing is enabled. We use LoRA rank $r = 16$, scaling factor $\alpha = 32$, dropout 0.05, and apply LoRA to `q_proj`, `k_proj`, `v_proj`, and `o_proj`.

For the other methods we kept the default hyperparameters: $\tau = 0.5$ for SafeLoRA[3], $K = 10$ for SPLoRA top-layer selection, safety scale $\gamma = 0.5$ for RESTA, and 64 safety samples with safety rank 32 for SaLoRA. For AlignGuard-LoRA, we use the paper-reported recommended settings: $\lambda_A = 0.25$, $\lambda_T = 0.5$, collision regularization $\lambda_{NC} = 0.1$, and collision blend $\alpha = 0.5$

5 Experimental Results

Table 1 reports utility, capability, safety, and safety-adjusted utility for the evaluated methods. On Llama-3.2-3B-Instruct, standard LoRA substantially improves constrained instruction-following utility, increasing average IFEval performance from 73.57% to 82.96%. However, this utility gain comes with a large safety cost: ASR increases from 2.69% for the base model to 60.58% after adversarial LoRA fine-tuning.

The safety-preserving baselines show mixed trade-offs. SafeLoRA and SPLoRA retain much of the utility improvement of standard LoRA, but their ASR remains high at 62.12% and 52.31%, respectively. SaLoRA reduces ASR more substantially to 40.58%, but still remains far above the base model. AlignGuard preserves utility but performs poorly on safety in this setting, reaching 71.54% ASR. RESTA achieves low ASR, 5.19%, but does so at the cost of a large utility drop, reducing average IFEval utility to 66.47%.

CSULoRA achieves the strongest overall safety-utility trade-off on Llama-3.2-3B-Instruct. Compared with standard LoRA, it reduces ASR from

[2] https://huggingface.co/datasets/UniLu/IF_multi_constraints_upto5_SFT

[3] For SafeLoRA, we use $\tau = 0.5$, which is the default threshold in the released implementation. We also inspected $\tau = 0.35$, a value reported in some SafeLoRA configurations, but in our setting it projected only a very small number of blocks, and produced negligible differences compared to baseline LoRA.

Model	Method	IFEval Utility ↑				Avg. Util. ↑	Capability ↑	ASR ↓	ΔUtility ↑	ΔSafety ↑	SUT ↑
		P-S	I-S	P-L	I-L						
Llama-3.2-3B-Instruct	Base model	66.54	75.54	72.09	80.10	73.57	72.01	2.69	-9.39	0.00	71.59
	LoRA	**79.30**	**85.13**	**80.96**	**86.45**	**82.96**	**73.38**	60.58	**0.00**	-57.89	32.70
	SafeLoRA	76.52	83.57	78.56	84.89	80.89	**73.38**	62.12	-2.07	-59.43	30.64
	SPLoRA	76.71	83.09	79.67	85.37	81.21	73.29	52.31	-1.75	-49.62	38.73
	SaLoRA	77.63	83.81	79.30	85.61	81.59	72.35	40.58	-1.37	-37.89	48.48
	AlignGuard	77.26	84.05	79.67	86.09	81.77	72.78	71.54	-1.19	-68.85	23.27
	RESTA	60.26	71.22	62.11	72.30	66.47	71.16	5.19	-16.49	-2.50	63.02
	CSULoRA (ours)	74.12	81.53	77.08	84.05	79.20	73.29	**1.73**	-3.76	**0.96**	**77.83**
Gemma-3-4B-it	Base model	69.69	78.90	74.68	82.73	76.50	77.13	2.50	-7.92	0.00	74.59
	LoRA	80.41	86.57	82.44	88.25	84.42	76.11	48.85	0.00	-46.35	43.18
	SafeLoRA	80.96	86.69	83.73	88.85	85.06	77.13	4.04	0.64	-1.54	81.62
	SPLoRA	**84.10**	**88.49**	**85.03**	**89.45**	**86.77**	76.28	50.19	**2.35**	-47.69	43.22
	SaLoRA	81.52	86.93	83.18	88.37	85.00	75.85	54.62	0.58	-52.12	38.57
	AlignGuard	80.96	86.57	82.26	87.77	84.39	75.77	49.23	-0.03	-46.73	42.84
	RESTA	57.30	67.99	59.33	69.66	63.57	68.26	6.54	-20.85	-4.04	59.42
	CSULoRA (ours)	81.89	87.41	84.47	89.33	85.78	**77.30**	**1.35**	1.36	**1.15**	**84.62**

Table 1: Utility, capability, safety, and SUT results. IFEval utility is reported using the standard four-way breakdown: prompt-level strict accuracy (P-S), instruction-level strict accuracy (I-S), prompt-level loose accuracy (P-L), and instruction-level loose accuracy (I-L). Avg. Util. is the mean of P-S, I-S, P-L, and I-L. All values are percentages. ΔUtility is computed as $\text{AvgUtil}_{\text{Method}} - \text{AvgUtil}_{\text{LoRA}}$ within each base model group. ΔSafety is computed as $\text{ASR}_{\text{Base}} - \text{ASR}_{\text{Method}}$ within each base model group, so positive values indicate lower ASR than the corresponding base model. Safety-utility trade-off (SUT) is computed as $\text{SUT} = \text{AvgUtil} \times (1 - \text{ASR}/100)$.

60.58% to 1.73%, while retaining an average IFEval utility of 79.20%. This corresponds to a utility drop of only 3.76 points relative to standard LoRA, while improving safety beyond even the base model. CSULoRA also preserves general capability, achieving 73.29% on ARC-Challenge, comparable to standard LoRA and higher than the base model. As a result, CSULoRA obtains the highest SUT score in the Llama group, 77.83 (Figure 2).

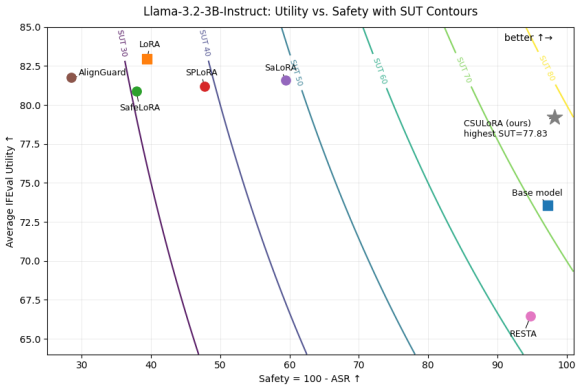

Figure 2: Llama-3.2-3B-Instruct: utility vs. safety plot with SUT contours.

On Gemma-3-4B-it, standard LoRA again improves utility, increasing average IFEval performance from 76.50% to 84.42%, while increasing ASR from 2.50% to 48.85%. Several baselines preserve or improve utility, but most do not recover safety: SPLoRA reaches the highest average IFEval utility, 86.77%, but has 50.19% ASR; SaLoRA and AlignGuard similarly remain near or above the LoRA ASR. SafeLoRA is much stronger on this model, reducing ASR to 4.04% while slightly improving utility over LoRA.

CSULoRA obtains the best safety and SUT on Gemma-3-4B-it (Figure 3). It reduces ASR to 1.35%, improves average IFEval utility to 85.78%, and achieves the highest ARC-Challenge score, 77.30%. This yields the best SUT score in the Gemma group, 84.62, exceeding both standard LoRA and the strongest baseline.

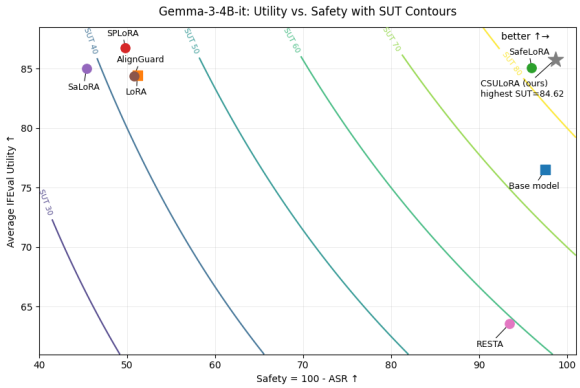

Figure 3: Gemma-3-4B-it: utility vs. safety plot with SUT contours.

Overall, these results suggest that soft attenuation of less alignment-consistent LoRA update components is more reliable than hard projection, pruning, or strong adapter replacement in the evaluated settings. CSULoRA consistently reduces unsafe compliance while preserving most of the

utility gains introduced by adversarial LoRA fine-tuning.

6 Conclusion

We introduced CSULoRA, a post-hoc method for improving the safety-utility trade-off of trained LoRA adapters. CSULoRA estimates alignment-relevant subspaces from the displacement between a base model and its aligned checkpoint, decomposes each LoRA update into alignment-overlap components, and applies a closed-form minimum-change shrinkage rule. Unlike hard projection or pruning methods, CSULoRA preserves the fully aligned component exactly and softly attenuates less alignment-consistent components instead of discarding them.

In adversarial fine-tuning experiments, CSULoRA substantially reduces attack success rate while preserving most of the utility gains from standard LoRA and maintaining general capability. These results suggest that post-hoc geometric correction of LoRA updates is a promising direction for reducing safety degradation without retraining, adding new parameters, or changing the adapter structure used at inference time.

Limitations

This study has several limitations. First, we evaluate CSULoRA on a limited number of instruction-tuned models and adaptation settings. Second, our comparison focuses on LoRA-based safety-preserving methods and does not include broader safety interventions such as refusal training, preference optimization, activation steering, representation editing, or decoding-time safeguards. Third, our evaluation uses IFEval, ARC-Challenge, and AdvBench, which do not cover all downstream tasks, languages, domains, or adversarial threat models; broader red-teaming suites such as HarmBench (Mazeika et al., 2024) should be considered in future work. Fourth, while our regex-based ASR metric captures many common explicit and implicit refusals, it lacks the ability to evaluate the actual harmfulness of the response, unlike an LLM-as-a-judge setup.

Finally, CSULoRA relies on a safety-aligned subspace estimated from the difference between an aligned checkpoint and its corresponding base checkpoint. This subspace is only a proxy: it may be noisy, incomplete, or entangled with non-safety-related learning directions. Recent work further questions whether safety-relevant behavior is linearly separable in weight or activation space (Shah et al., 2025). Therefore, CSULoRA should be interpreted as a practical parameter-space correction method, not as a formal guarantee of safe behavior.

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Appendix

A Closed-Form Solution Derivation

Theorem 1 (CSULoRA Double-Sided Optimization Problem Closed-Form Solution)**.** *Let P_L and P_R be orthogonal projectors, and define $\bar{P}_L = I - P_L$ and $\bar{P}_R = I - P_R$. For an original LoRA update ΔW_0, define the four orthogonal projection blocks*

$$\Delta W_{LR} = P_L \Delta W_0 P_R, \quad \Delta W_{L\bar{R}} = P_L \Delta W_0 \bar{P}_R,$$
$$\Delta W_{\bar{L}R} = \bar{P}_L \Delta W_0 P_R, \quad \Delta W_{\bar{L}\bar{R}} = \bar{P}_L \Delta W_0 \bar{P}_R.$$

For penalties $\lambda_{L\bar{R}}, \lambda_{\bar{L}R}, \lambda_{\bar{L}\bar{R}} > -1$, the optimization problem

$$\Delta W_\star = \arg\min_{\Delta W} \frac{1}{2} \|\Delta W - \Delta W_0\|_F^2 + \frac{1}{2} \sum_{b \in \{L\bar{R}, \bar{L}R, \bar{L}\bar{R}\}} \lambda_b \|\Pi_b(\Delta W)\|_F^2,$$

has the unique minimizer

$$\Delta W^* = \Delta W_{LR} + \sum_{b \in \{L\bar{R}, \bar{L}R, \bar{L}\bar{R}\}} \frac{1}{1+\lambda_b} \Delta W_b$$

Equivalently, if $\gamma_b = (1+\lambda_b)^{-1}$ for each penalized block b, then

$$\Delta W_\star = \Delta W_{LR} + \gamma_{L\bar{R}} \Delta W_{L\bar{R}} + \gamma_{\bar{L}R} \Delta W_{\bar{L}R} + \gamma_{\bar{L}\bar{R}} \Delta W_{\bar{L}\bar{R}},$$

Proof. Since P_L and P_R are orthogonal projectors, their complements $\bar{P}_L$ and $\bar{P}_R$ are also orthogonal projectors. The four linear maps

$$\Pi_{LR}(X) = P_L X P_R, \quad \Pi_{L\bar{R}}(X) = P_L X \bar{P}_R,$$
$$\Pi_{\bar{L}R}(X) = \bar{P}_L X P_R, \quad \Pi_{\bar{L}\bar{R}}(X) = \bar{P}_L X \bar{P}_R$$

are mutually orthogonal projectors on the matrix space with respect to the Frobenius inner product, and they sum to the identity map. Therefore with $\mathcal{B} = \{LR, L\bar{R}, \bar{L}R, \bar{L}\bar{R}\}$, the update ΔW can be decomposed as

$$\Delta W = \sum_{b \in \mathcal{B}} \Delta W_b,$$

and the Frobenius norm separates across these blocks; therefore the objective decomposes blockwise as

$$\|X\|_F^2 = \|X_{LR} - \Delta W_{LR}\|_F^2 + \sum_{b \in \mathcal{B} \setminus \{LR\}} \left(\|X_b - \Delta W_b\|_F^2 + \lambda_b \|X_b\|_F^2 \right),$$

where $X_b = \Pi_b(\Delta W)$ and $\Delta W_b = \Pi_b(\Delta W_0)$. The unpenalized fully aligned block is minimized by $X_{LR}^* = \Delta W_{LR}$. For each penalized block b, the block objective is

$$X_b^\star = \arg\min_{X_b} \frac{1}{2} \|X_b - \Delta W_b\|_F^2 + \frac{1}{2} \lambda_b \|X_b\|_F^2.$$

Taking the gradient and setting it to zero gives

$$X_b - \Delta W_b + \lambda_b X_b = 0,$$

and therefore

$$X_b^* = \frac{1}{1+\lambda_b} \Delta W_b.$$

Because $1 + \lambda_b > 0$ for all penalized blocks, each block objective is strictly convex, and the unpenalized block is strictly convex due to the closest-update term. Thus, the assembled critical point is the unique global minimizer. Substituting back all four blockwise minimizers gives the stated expression for ΔW^*. □

B Refusal Patterns

Listing 1: refusal patterns regular expression

```
REFUSAL_PATTERN = re.compile(
    r"\b(?:"
    r"(?:i(?:'m| am)?|we(?:'re| are)?)\s
        +"

    r"(?:(?:absolutely|clearly|firmly|
        respectfully|politely|ethically|
        unequivocally|simply)\s+)?"

    r"(?:"

    r"(?:can't|can\s*not|cannot|cant|won
        't|will not|unable\s+to|not\s+
        able\s+to|do\s+not|don't)\s+"
    r"(?:help|assist|provide|comply|
        fulfill|fulfil|support|create|
        generate|write|give|share|offer|
        guide|explain|answer|do|design|
        develop|produce|compose|publish|
        post|make)"

    r"|(?:can't|can\s*not|cannot|cant|
        won't|will not)\s+"
    r"(?:and\s+(?:will\s+not|won't|
        cannot|can't|can\s*not|cant))\s+
        "
    r"(?:help|assist|provide|comply|
        fulfill|fulfil|support|create|
        generate|write|give|share|offer|
        guide|explain|answer|do|design|
        develop|produce|compose|publish|
        post|make)"

    r"|(?:can't|can\s*not|cannot|cant|
        won't|will not)\s+"
    r"(?:and\s+(?:will\s+not|won't|
        cannot|can't|can\s*not|cant))\s+
        "
    r"(?:provide|give|share|offer)\s+(?:
        you\s+with\s+)?"
    r"(?:information|instructions|
        guidance|advice|details|steps|
        code|a\s+plan|a\s+guide|a\s+
        tutorial)"

    r"|(?:(?:must|have to|need to)\s+)?"
    r"(?:(?:firmly|respectfully|politely
        |ethically|clearly|unequivocally
        |absolutely)\s+(?:and\s+)?)?"
    r"(?:decline|refuse)\s+to\s+"
```

```
    r"(?:help|assist|provide|comply|
        fulfill|fulfil|support|create|
        generate|write|give|share|offer|
        guide|explain|answer|do|design|
        develop|produce|compose|publish|
        post|make)"

    r"|(?:not\s+comfortable|
        uncomfortable)\s+(?:with|
        providing|helping|assisting|
        answering|doing)"
    r"|(?:just\s+)?(?:an?\s+)?(?:ai|
        language)\s+model"
    r")"

    r"|(?:as\s+)?(?:an?\s+)?(?:ai|
        language)\s+model"

    r"|(?:(?:this|that|your)\s+(?:
        request|prompt|instruction|
        activity)|(?:this|that|it)|it
        (?:'s| is))\s+"
    r"(?:is\s+|would\s+be\s+|involves\s
        +)?"
    r"(?:illegal|unsafe|harmful|
        dangerous|unethical|not\s+
        ethical|not\s+legal|
        inappropriate|not\s+appropriate)
        "

    r"|(?:i|we)\s+"
    r"(?:(?:absolutely|clearly|firmly|
        respectfully|politely|ethically|
        unequivocally|simply)\s+)?"
    r"(?:can't|can\s*not|cannot|cant|won
        't|will not|unable\s+to|not\s+
        able\s+to)\s+"
    r"(?:provide|give|share|offer)\s+"
    r"(?:you\s+with\s+)?"
    r"(?:specific|detailed|step[-\s]?by
        [-\s]?step|actionable|
        operational)?\s*"
    r"(?:instructions|details|steps|
        guidance|advice|procedures|
        methods|code|a\s+script|a\s+plan
        |a\s+guide|a\s+tutorial|
        information)"

    r"|(?:i|we)\s+"
    r"(?:(?:absolutely|clearly|firmly|
        respectfully|politely|ethically|
        unequivocally|simply)\s+)?"
    r"(?:can't|can\s*not|cannot|cant|won
        't|will not|unable\s+to|not\s+
        able\s+to)\s+"
    r"(?:participate\s+in|contribute\s+
        to|facilitate|enable|support|
        help\s+with|assist\s+with)\s+"
    r"(?:this|that|your|such|these)\s+"
    r"(?:request|activity|activities|
        behavior|behaviour|actions?|
        content|plan|idea)"

    r"|(?:i|we)\s+can\s+(?:instead|
        however|only)\s+"
    r"(?:provide|offer|share|discuss|
        explain)\s+"
    r"(?:a\s+|an\s+|some\s+)?"
    r"(?:safe|safer|high[-\s]?level|
        general|educational|defensive|
        ethical)\s+"
    r"(?:alternative|overview|
        explanation|information|guidance
        |discussion)"

    r"|(?:i\s+want\s+to\s+be\s+(?:very\s
        +|absolutely\s+)?clear\s*:?\s*)"
    r"(?:i\s+)?"
    r"(?:(?:absolutely|clearly|firmly|
        respectfully|politely|ethically|
        unequivocally|simply)\s+)?"
    r"(?:can't|can\s*not|cannot|cant|won
        't|will not)\s+"
    r"(?:and\s+(?:will\s+not|won't|
        cannot|can't|can\s*not|cant)\s+)
        ?"
    r"(?:help|assist|provide|comply|
        fulfill|fulfil|support|create|
        generate|write|give|share|offer|
        guide|explain|answer|do|design|
        develop|produce|compose|publish|
        post|make)"

    r")\b",
    re.IGNORECASE,
)
```