ON SATURATED PARTITIONS

Abstract

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Let L be a language over a vocabulary V and let denote by E (V) the set of all equivalence relations (partitions) on V. If $\rho \in E(V)$ and $x \in V$ then we shall denote by $\rho(x)$ the cell of ρ containing the element x.

<u>Definition 1.</u> A partition $\rho \in E(V)$ is said to be saturated if for every marked ρ-structure $\rho(x_1)\,\rho(x_2)\ldots\rho(x_n)$ and for every i $(1 \leq i \leq n)$ there exist such elements x_j' (j=1,..., i-1, i+1,...,n) that $x_j' \in \rho(x_j)$ and $x_1' \ldots x_{i-1}'\, x_i\, x_{i+1}' \ldots x_n' \in L$.

Our purpose is to find (in the case of a finite vocabulary) the greatest saturated partition Z of the language.

<u>Definition 2.</u> Let $\rho \in E(V)$ and $x, y \in V$. We shall say that x ρ-dominates y $(x \xrightarrow{\rho} y)$ if for every string $x_1 \ldots x_n \in L$ where $x_i = x$ there exist such elements x_j' (j=1,...,i-1, i+1,...,n) that $x_j' \,\rho\, x_j$ and $x_1' \ldots x_{i-1}'\, y\, x_{i+1}' \ldots x_n' \in L$.

We can introduce now the partition ρ^*, called the asterisk of the partition ρ : $x \rho^* y$ if both $x \xrightarrow{\rho} y$ and $y \xrightarrow{\rho} x$ hold true.

<u>Theorem 1.</u> ρ is a saturated partition if and only if ρ is finer than ρ^*.

The connection between the asterisk and the derivative of a partition is given by:

<u>Theorem 2.</u> In order that $\rho' = \rho^*$ it is necessary and sufficient that ρ be saturated.

By using the notation: $\rho^{0*} = \rho$ and $\rho^{(n+1)*} = (\rho^{n*})^*$ we have :

<u>Theorem 3.</u> There exists a natural number n so that $Z = \infty^{n*}$ (where ∞ is the improper partition of V).