A COMPUTATIONAL THEORY OF DISPOSITIONS

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ABSTRACT

Informally, a *disposition* is a proposition which is preponderantly, but no necessarily always, true. For example, *birds can fly* is a disposition, as are the propositions *Swedes are blond* and *Spaniards are dark*.

An idea which underlies the theory described in this paper is that a disposition may be viewed as a proposition with implicit fuzzy quantifiers which are approximations to *all* and *always*, e.g., *almost all, almost always, most, frequently,* etc. For example, *birds can fly* may be interpreted as the result of supressing the fuzzy quantifier *most* in the proposition *most birds can fly*. Similarly, *young men like young women* may be read as *most young men like mostly young women*. The process of transforming a disposition into a proposition is referred to as *explicitation* or *restoration*.

Explicitation sets the stage for representing the meaning of a proposition through the use of test-score semantics (Zadeh, 1978, 1982). In this approach to semantics, the meaning of a proposition, *p*, is represented as a procedure which tests, scores and aggregates the elastic constraints which are induced by *p*.

The paper closes with a description of an approach to reasoning with dispositions which is based on the concept of a fuzzy syllogism. Syllogistic reasoning with dispositions has an important bearing on commonsense reasoning as well as on the management of uncertainty in expert systems. As a simple application of the techniques described in this paper, we formulate a definition of *typicality* -- a concept which plays an important role in human cognition and is of relevance to default reasoning.

1. Introduction

Informally, a disposition is a proposition which is preponderantly, but not necessarily always, true. Simple examples of dispositions are: *Smoking is addictive, exercise is good for your health, long sentences are more difficult to parse than short sentences, overeating causes obesity, Trudi is always right,* etc. Dispositions play a central role in human reasoning, since much of human knowledge and, especially, commonsense knowledge, may be viewed as a collection of dispositions.

The concept of a disposition gives rise to a number of related concepts among which is the concept of a *dispositional predicate*. Familiar examples of unary predicates of this type are: *Healthy, honest, optimist, safe,* etc., with binary dispositional predicates exemplified by: *taller than* in *Swedes are taller than Frenchmen, like* in *Italians are like Spaniards, like* in *young men like young women,* and *smokes* in *Ron smokes cigarettes.* Another related concept is that of a *dispositional command* (or *imperative*) which is exemplified by *proceed with caution, avoid overexertion, keep under refrigeration, be frank,* etc.

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The basic idea underlying the approach described in this paper is that a disposition may be viewed as a proposition with suppressed, or, more generally, implicit fuzzy quantifiers such as *most, almost all, almost always, usually, rarely, much of the time,* etc[1]. To illustrate, the disposition *overating causes obesity* may be viewed as the result of suppression of the fuzzy quantifier *most* in the proposition *most of those who overeat are obese.* Similarly, the disposition *young men like young women* may be interpreted as *most young men like mostly young women.* It should be stressed, however, that *restoration* (or *explicitation*) -- viewed as the inverse of suppression -- is an interpretation-dependent process in the sense that, in general, a disposition may be interpreted in different ways depending on the manner in which the fuzzy quantifiers are restored and defined.

The implicit presence of fuzzy quantifiers stands in the way of representing the meaning of dispositional concepts through the use of conventional methods based on truth-conditional, possible-world or model-theoretic semantics (Cresswell, 1973; McCawley, 1981; Miller and Johnson-Laird, 1976). In the computational approach which is described in this paper, a fuzzy quantifier is manipulated as a fuzzy number. This idea serves two purposes. First, it provides a basis for representing the meaning of dispositions; and second, it opens a way of reasoning with dispositions through the use of a collection of syllogisms. This aspect of the concept of a disposition is of relevance to default reasoning and non-monotonic logic (McCarthy, 1980; McDermott and Doyle, 1980; McDermott, 1982; Reiter, 1983).

To illustrate the manner in which fuzzy quantifiers may be manipulated as fuzzy numbers, assume that, after restoration, two dispositions d_1 and d_2 may be expressed as propositions of the form

$$p_1 \triangleq Q_1 \, A's \text{ are } B's \qquad (1.1)$$

$$p_2 \triangleq Q_2 \, B's \text{ are } C's \, , \qquad (1.2)$$

in which Q_1 and Q_2 are fuzzy quantifiers, and A, B and C are fuzzy predicates. For example,

$$p_1 \triangleq \textit{most students are undergraduates} \qquad (1.3)$$

$$p_2 \triangleq \textit{most undergraduates are young} \, .$$

By treating p_1 and p_2 as the major and minor premises in a syllogism, the following *chaining* syllogism may be established if $B \subset A$ (Zadeh, 1983):

1. In the literature of linguistics, logic and philosophy of languages, fuzzy quantifiers are usually referred to as *vague* or *generalized* quantifiers (Barwise and Cooper, 1981; Peterson, 1979). In the approach described in this paper, a fuzzy quantifier is interpreted as a fuzzy number which provides an approximate characterization of absolute or relative cardinality.

$$Q_1\ A's\ are\ B's \qquad (1.4)$$
$$\underline{Q_2\ B's\ are\ C's}$$
$$\geq(Q_1 \otimes Q_2)\ A's\ are\ C's$$

in which $Q_1 \otimes Q_2$ represents the product of the fuzzy numbers Q_1 and Q_2 (Figure 1).

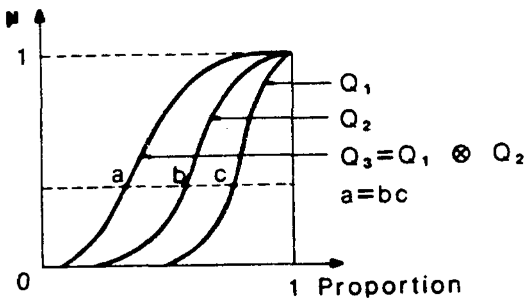

Figure 1. Multiplication of fuzzy quantifiers

and $\geq(Q_1 \otimes Q_2)$ should be read as "at least $Q_1 \otimes Q_2$." As shown in Figure 1, Q_1 and Q_2 are defined by their respective possibility distributions, which means that if the value of Q_1 at the point u is α, then α represents the possibility that the proportion of *A's* in *B's* is u.

In the special case where p_1 and p_2 are expressed by (1.3), the chaining syllogism yields

most students are undergraduates

most undergraduates are young

most^2 *students are young*

where *most*^{2} represents the product of the fuzzy number *most* with itself (Figure 2).

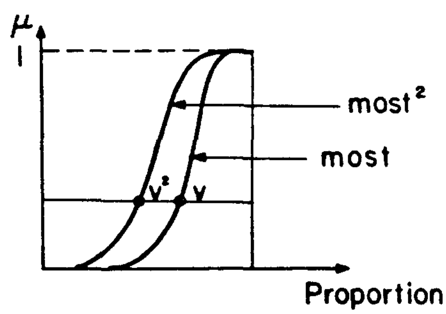

Figure 2. Representation of *most* and *most*^{2}.

2. Meaning Representation and Test-Score Semantics

To represent the meaning of a disposition, d, we employ a two-stage process. First, the suppressed fuzzy quantifiers in d are restored, resulting in a fuzzily quantified proposition p. Then, the meaning of p is represented -- through the use of test-score semantics (Zadeh, 1978, 1982) -- as a procedure which acts on a collection of relations in an explanatory database and returns a test score which represents the degree of compatibility of p with the database. In effect, this implies that p may be viewed as a collection of elastic constraints which are tested, scored and aggregated by the meaning-representation procedure. In test-score semantics, these elastic constraints play a role which is analogous to that truth-conditions in truth-conditional semantics (Cresswell, 1973).

As a simple illustration, consider the familiar example

$$d \triangleq \textit{snow is white}$$

which we interpret as a disposition whose intended meaning is the proposition

$$p \triangleq \textit{usually snow is white}\ .$$

To represent the meaning of p, we assume that the *explanatory database*, *EDF* (Zadeh, 1982), consists of the following relations whose meaning is presumed to be known

$$EDF \triangleq WHITE\,[Sample;\mu] + USUALLY[Proportion;\mu]\ ,$$

in which + should be read as *and*. The ith row in *WHITE* is a tuple (S_i, r_i), $i = 1,...,m$, in which S_i is the ith sample of snow, and r_i is is the degree to which the color of S_i matches white. Thus, r_i may be interpreted as the test score for the constraint on the color of S_i induced by the elastic constraint *WHITE*. Similarly, the relation *USUALLY* may be interpreted as an elastic constraint on the variable *Proportion*, with μ representing the test score associated with a numerical value of *Proportion*.

The steps in the procedure which represents the meaning of p may be described as follows:

1. Find the proportion of samples whose color is white:

$$\rho = \frac{r_1 + \cdots + r_m}{m}$$

in which the proportion is expressed as the arithmetic average of the test scores.

2. Compute the degree to which ρ satisfies the constraint induced by *USUALLY*:

$$\tau = {}_{\mu}\ USUALLY[Proportion = \rho]\ ,$$

in which τ is the overall test score, i.e., the degree of compatibility of p with *ED*, and the notation ${}_{\mu}R\,[X = a]$ means: Set the variable X in the relation R equal to a and read the value of the variable μ.

More generally, to represent the meaning of a disposition it is necessary to define the cardinality of a fuzzy set. Specifically, if A is a subset of a finite universe of discourse $U = \{u_1,...,u_n\}$, then the *sigma-count* of A is defined as

$$\Sigma Count(A) = \Sigma_i \mu_A(u_i)\ , \qquad (2.1)$$

in which $\mu_A(u_i)$, $i = 1,...,n$, is the grade of membership of u_i in A (Zadeh, 1983a), and it is understood that the sum may be rounded, if need be, to the nearest integer. Furthermore, one may stipulate that the terms whose grade of membership falls below a specified threshold be excluded from the summation. The purpose of such an exclusion is to avoid a situation in which a large number of terms with low grades of membership become count-equivalent to a small number of terms with high membership.

The *relative sigma-count*, denoted by $\Sigma Count(B/A)$, may be interpreted as the proportion of elements of B in A. More explicitly,

$$\Sigma Count(B/A) = \frac{\Sigma Count(A \cap B)}{\Sigma Count(A)}\ , \qquad (2.2)$$

where $B \cap A$, the intersection of B and A, is defined by

$$\mu_{B \cap A}(u) = \mu_B(u) \wedge \mu_A(u) , u \in U ,$$

where $\wedge$ denotes the min operator in infix form. Thus, in terms of the membership functions of B and A, the relative sigma-count of B and A is given by

$$\Sigma Count(B/A) = \frac{\Sigma_i \mu_B(u_i) \wedge \mu_A(u_i)}{\Sigma_i \mu_A(u_i)} . \qquad (2.3)$$

As an illustration, consider the disposition

$$d \triangleq \textit{overating causes obesity} \qquad (2.4)$$

which after restoration is assumed to read[2]

$$p \triangleq \textit{most of those who overeat are obese} . \qquad (2.5)$$

To represent the meaning of p, we shall employ an explanatory database whose constituent relations are:

$$EDF \triangleq POPULATION[Name; Overeat; Obese]$$
$$+ MOST[Proportion;\mu] .$$

The relation *POPULATION* is a list of names of individuals, with the variables *Overeat* and *Obese* representing, respectively, the degrees to which *Name* overeats and is obese. In *MOST*, μ is the degree to which a numerical value of *Proportion* fits the intended meaning of *MOST*.

To test procedure which represents the meaning of p involves the following steps.

1. Let $Name_i$, $i = 1,...,m$, be the name of ith individual in *POPULATION*. For each $Name_i$, find the degrees to which $Name_i$ overeats and is obese:

$$\alpha_i \triangleq \mu_{OVEREAT}(Name_i) \triangleq {}_{Overeat}POPULATION[Name = Name_i]$$
$$\beta_i \triangleq \mu_{OBESE}(Name_i) = {}_{Obese}POPULATION[Name = Name_i] .$$

2. Compute the relative sigma-count of *OBESE* in *OVEREAT*:

$$\rho \triangleq \Sigma Count(OBESE/OVEREAT) = \frac{\Sigma_i \alpha_i \wedge \beta_i}{\Sigma_i \alpha_i} .$$

3. Compute the test score for the constraint induced by *MOST*:

$$\tau = {}_{\mu}MOST[Proportion = \rho] .$$

This test score represents the compatibility of p with the explanatory database.

3. The Scope of a Fuzzy Quantifier

In dealing with the conventional quantifiers *all* and *some* in first-order logic, the scope of a quantifier plays an essential role in defining its meaning. In the case of a fuzzy quantifier which is characterized by a relative sigma-count, what matters is the identity of the sets which enter into the relative count. Thus, if the sigma-count is of the form $\Sigma Count(B/A)$, which should be read as the proportion of B's in A's, then B and A will be referred to as the *n-set* (with n standing for numerator) and *b-set* (with b standing for base), respectively. The ordered pair {n-set, b-set}, then, may be viewed as a generalization of the concept of the scope of a quantifier. Note, however, that, in this sense, the scope of a fuzzy quantifier is a semantic rather than syntactic concept.

As a simple illustration, consider the proposition $p \triangleq$ ***most students are undergraduates***. In this case, the n-set of ***most*** is ***undergraduates***, the b-set is ***students***, and the scope of ***most*** is the pair {***undergraduates, students***}.

2. It should be understood that (2.5) is just one of many possible interpretations of (2.4), with no implication that is constitutes a prescriptive interpretation of causality. See Suppes (1970).

As an additional illustration of the interaction between scope and meaning, consider the disposition

$$d \triangleq \textit{young men like young women} . \qquad (3.1)$$

Among the possible interpretations of this disposition, we shall focus our attention on the following (the symbol rd denotes a restoration of a disposition):

$$rd_1 \triangleq \textit{most young men like most young women}$$
$$rd_2 \triangleq \textit{most young men like mostly young women} .$$

To place in evidence the difference between rd_1 and rd_2, it is expedient to express them in the form

$$rd_1 = \textit{most young men } P_1$$
$$rd_2 = \textit{most young men } P_2 ,$$

where P_1 and P_2 are the fuzzy predicates

$$P_1 \triangleq \textit{likes most young women}$$

and

$$P_2 \triangleq \textit{likes mostly young women} ,$$

with the understanding that, for grammatical correctness, ***likes*** in P_1 and P_2 should be replaced by ***like*** when P_1 and P_2 act as constituents of rd_1 and rd_2. In more explicit terms, P_1 and P_2 may be expressed as

$$P_1 \triangleq P_1[Name;\mu] \qquad (3.2)$$
$$P_2 \triangleq P_2[Name;\mu] ,$$

in which *Name* is the name of a male person and μ is the degree to which the person in question satisfies the predicate. (Equivalently, μ is the grade of membership of the person in the fuzzy set which represents the denotation or, equivalently, the extension of the predicate.)

To represent the meaning of P_1 and P_2 through the use of test-score semantics, we assume that the explanatory database consists of the following relations (Zadeh, 1983b):

$$EDF \triangleq POPULATION[Name; Age; Sex] +$$
$$LIKE[Name1;Name2; \mu] + YOUNG[Age; \mu] +$$
$$MOST[Proportion; \mu] .$$

In *LIKE*, μ is the degree to which *Name1* likes *Name2*; and in *YOUNG*, μ is the degree to which a person whose age is *Age* is young.

First, we shall represent the meaning of P_1 by the following test procedure.

1. Divide *POPULATION* into the population of males, *M.POPULATION*, and the population of females, *F.POPULATION*:

$$M.POPULATION \triangleq {}_{Name,Age}POPULATION[Sex=Male]$$
$$F.POPULATON \triangleq {}_{Name,Age}POPULATION[Sex=Female] ,$$

where ${}_{Name,Age}POPULATION$ denotes the projection of *POPULATION* on the attributes *Name* and *Age*.

2. For each $Name_j$, $j = 1,...,L$, in *F.POPULATION*, find the age of $Name_j$:

$$A_j \triangleq {}_{Age}F.POPULATION[Name=Name_j] .$$

3. For each $Name_j$, find the degree to which $Name_j$ is young:

$$\alpha_j \triangleq {}_{\mu}YOUNG[Age=A_j] ,$$

where α_j may be interpreted as the grade of membership of $Name_j$ in the fuzzy set, YW, of young women.

4. For each $Name_i$, $i=1,...,K$, in $M.POPULATION$, find the age of $Name_i$:

$$B_i \triangleq {}_{Age}M.POPULATION[Name{=}Name_i] .$$

5. For each $Name_j$, find the degree to which $Name_i$ likes $Name_j$:

$$\beta_{ij} \triangleq {}_{\mu}LIKE[Name1 = Name_i;Name2 = Name_j] ,$$

with the understanding that β_{ij} may be interpreted as the grade of membership of $Name_j$ in the fuzzy set, WL_i, of women whom $Name_i$ likes.

6. For each $Name_j$ find the degree to which $Name_i$ likes $Name_j$ and $Name_j$ is young:

$$\gamma_{ij} \triangleq \alpha_j \wedge \beta_{ij} .$$

Note: As in previous examples, we employ the aggregation operator min ($\wedge$) to represent the meaning of conjunction. In effect, γ_{ij} is the grade of membership of $Name_j$ in the intersection of the fuzzy sets WL_i and YW.

7. Compute the relative sigma-count of women whom $Name_i$ likes among young women:

$$\rho_i \triangleq \Sigma Count(WL_i/YW) \tag{3.4}$$
$$= \frac{\Sigma Count(WL_i \cap YW)}{\Sigma Count(YW)}$$
$$= \frac{\Sigma_j \gamma_{ij}}{\Sigma \alpha_j}$$
$$= \frac{\Sigma_j \alpha_j \wedge \beta_{ij}}{\Sigma_j \alpha_j} .$$

8. Compute the test score for the constraint induced by $MOST$:

$$\tau_i = {}_{\mu} MOST[Proportion = \rho_i] \tag{3.5}$$

This test-score way be interpreted as the degree to which $Name_i$ satisfies P_1, i.e.,

$$\tau_i = {}_{\mu} P_1[Name = Name_i]$$

The test procedure described above represents the meaning of P_1. In effect, it tests the constraint expressed by the proposition

$$\Sigma Count(YW/WL_i) \text{ is } MOST$$

and implies that the n-set and the b-set for the quantifier ***most*** in P_1 are given by:

$$n\text{-}set = WL_i = {}_{Name2}LIKE[Name1 = Name_i] \cap F.POPULATION$$

and

$$b\text{-}set = YW = YOUNG \cap F.POPULATION .$$

By contrast, in the case of P_2, the identities of the n-set and the b-set are interchanged, i.e.,

$$n\text{-}set = YW$$

and

$$b\text{-}set = WL_i ,$$

which implies that the constraint which defines P_2 is expressed by

$$\Sigma Count(YW/WL_i) \text{ is } MOST .$$

Thus, whereas the scope of the quantifier ***most*** in P_1 is $\{WL_i, YW\}$, the scope of ***mostly*** in P_2 is $\{YW, WL_i\}$.

Having represented the meaning of P_1 and P_2, it becomes a simple matter to represent the meaning of rd, and rd_2. Taking rd_1, for example, we have to add the following steps to the test procedure which defines P_1.

9. For each $Name_i$, find the degree to which $Name_i$ is young:

$$\delta_i \triangleq {}_{\mu}YOUNG[Age = B_i] ,$$

where δ_i may be interpreted as the grade of membership of $Name_i$ in the fuzzy set, YM, of young men.

10. Compute the relative sigma-count of men who have property P_1 among young men:

$$\delta \triangleq \Sigma Count(P_1/YM)$$
$$= \frac{\Sigma Count(P_i \cap YM)}{\Sigma Count(YM)}$$
$$= \frac{\Sigma_i \tau_i \wedge \delta_i}{\Sigma_i \delta_i} .$$

11. Test the constraint induced by $MOST$:

$$\tau = {}_{\mu}MOST[Proportion{=}\rho] .$$

The test score expressed by (3.6) represents the overall test score for the disposition

$$d \triangleq \textit{young men like young women}$$

if d is interpreted as rd_1. If d is interpreted as rd_2, which is a more likely interpretation, then the procedure is unchanged except that τ_i in (3.5) should be replaced by

$$\tau_i = {}_{\mu}MOST[Proportion = \delta_i]$$

where

$$\delta_i \triangleq \Sigma Count(YW/WL_i)$$
$$= \frac{\Sigma_j \alpha_i \wedge \beta_{ij}}{\Sigma_j \beta_{ij}} .$$

4. Representation of Dispositional Commands and Concepts

The approach described in the preceding sections can be applied not only to the representation of the meaning of dispositions and dispositional predicates, but, more generally, to various types of semantic entities as well as dispositional concepts.

As an illustration of its application to the representation of the meaning of dispositional commands, consider

$$dc \triangleq \textit{stay away from bald men} , \tag{4.1}$$

whose explicit representation will be assumed to be the command

$$c \triangleq \textit{stay away from most bald men} . \tag{4.2}$$

The meaning of c is defined by its compliance criterion (Zadeh, 1982) or, equivalently, its propositional content (Searle, 1979), which may be expressed as

$$cc \triangleq \textit{staying away from most bald men} .$$

To represent the meaning of cc through the use of test-score semantics, we shall employ the explanatory database

$$EDF \triangleq RECORD[Name;\ \mu Bald;\ Action]$$
$$+\ MOST[Proposition;\ \mu]\ .$$

The relation *RECORD* may be interpreted as a diary -- kept during the period of interest -- in which *Name* is the name of a man; $\mu Bald$ is the degree to which he is bald; and *Action* describes whether the man in question was stayed away from (Action=1) or not (Action=0).

The test procedure which defines the meaning of *dc* may be described as follows:

1. For each $Name_i$, $i=1,...,n$, find (a) the degree to which $Name_i$ is bald; and (b) the action taken:

$$\mu Bald_i \triangleq {}_{\mu Bald}RECORD[Name = Name_i]$$
$$Action_i \triangleq {}_{Action}RECORD[Name = Name_i]\ .$$

2. Compute the relative sigma-count of compliance:

$$\rho = \frac{1}{n}\left(\Sigma_i\ \mu Bald_i \wedge Action_i\right)\ . \qquad (4.3)$$

3. Test the constraint induced by *MOST*:

$$\tau = {}_{\mu}MOST[Proposition = \rho]\ . \qquad (4.4)$$

The computed test score expressed by (4.4) represents the degree of compliance with *c*, while the procedure which leads to τ represents the meaning of *dc*.

The concept of dispositionality applies not only to semantic entities such as propositions, predicates, commands, etc., but, more generally, to concepts and their definitions. As an illustration, we shall consider the concept of typicality -- a concept which plays a basic role in human reasoning, especially in default reasoning (Reiter, 1983), concept formation (Smith and Medin, 1981), and pattern recognition (Zadeh, 1977).

Let U be a universe of discourse and let A be a fuzzy set in A (e.g., $U \triangleq$ ***cars*** and $A \triangleq$ ***station wagons***). The definition of a ***typical*** element of A may be expressed in verbal terms as follows:

t is a ***typical*** element of A if and only if (4.5)

(a) t has a high grade of membership in A, and

(b) most elements of A are similar to t.

It should be remarked that this definition should be viewed as a ***dispositional definition***, that is, as a definition which may fail, in some cases, to reflect our intuitive perception of the meaning of typicality.

To put the verbal definition expressed by (4.5) into a more precise form, we can employ test-score semantics to represent the meaning of (a) and (b). Specifically, let S be a similarity relation defined on U which associates with each element u in U the degree to which u is similar to t[3]. Furthermore, let $S(t)$ be the ***similarity class*** of t, i.e., the fuzzy set of elements of U which are similar to t. What this means is that the grade of membership of u in $S(t)$ is equal to $\mu_S(t,u)$, the degree to which u is similar to t (Zadeh, 1971).

Let *HIGH* denote the fuzzy subset of the unit interval which is the extension of the fuzzy predicate ***high***. Then, the verbal definition (4.5) may be expressed more precisely in the form:

t is a ***typical*** element of A if and only if (4.6)

(a) $\mu_A(t)$ is *HIGH*

(b) $\Sigma Count(S(t)/A)$ is *MOST*.

The fuzzy predicate ***high*** may be characterized by its membership function μ_{HIGH} or, equivalently, as the fuzzy relaton $HIGH\ [Grade;\ \mu]$, in which *Grade* is a number in the interval [0,1] and μ is the degree to which the value of *Grade* fits the intended meaning of ***high***.

An important implication of this definition is that typicality is a matter of degree. Thus, it follows at once from (4.6) that the degree, τ, to which t is typical or, equivalently, the grade of membership of t in the fuzzy set of typical elements of A, is given by

$$\tau = {}_{\mu}HIGH[Grade = t] \wedge \qquad (4.7)$$
$${}_{\mu}MOST[Proportion = \Sigma Count(S(t)/A]\ .$$

In terms of the membership functions of *HIGH, MOST*, S and A, (4.7) may be written as

$$\tau = \mu_A(t) \wedge \mu_{MOST}\left[\frac{\Sigma_u\ \mu_S(t,u) \wedge \mu_A(u)}{\Sigma_u\ \mu_A(u)}\right], \qquad (4.8)$$

where μ_{HIGH}, μ_{MOST}, μ_S and μ_A are the membership functions of *HIGH, MOST, S* and *A*, respectively, and the summation Σ_u extends over the elements of U.

It is of interest to observe that if $\mu_A(t) = 1$ and

$$\mu_S(t,u) = \mu_A(u)\ , \qquad (4.9)$$

that is, the grade of membership of u in A is equal to the degree of similarity of u to t, then the degree of typicality of t is unity. This is reminiscent of definitions of prototypicality (Rosch, 1978) in which the grade of membership of an object in a category is assumed to be inversely related to its "distance" from the prototype.

In a definition of prototypicality which we gave in Zadeh (1982), a prototype is interpreted as a so-called σ ***-summary***. In relation to the definition of typicality expressed by (4.5), we may say that a prototype is a σ ***-summary*** of typical elements of A. In this sense, a prototype is ***not***, in general, an element of U whereas a typical element of A is, by definition, an element of U. As a simple illustration of this difference, assume that U is a collection of movies, and A is the fuzzy set of Western movies. A prototype of A is a summary of the summaries (i.e., plots) of Western movies, and thus is not a movie. A typical Western movie, on the other hand, is a movie and thus is an element of U.

5. Fuzzy Syllogisms

A concept which plays an essential role in reasoning with dispositions is that of a ***fuzzy syllogism*** (Zadeh, 1983c). As a general inference schema, a fuzzy syllogism may be expressed in the form

$$\begin{array}{l} Q_1\ A's\ are\ B's \\ \underline{Q_2\ C's\ are\ D's} \\ ?Q_3\ E's\ are\ F's \end{array} \qquad (5.1)$$

where Q_1 and Q_2 are given fuzzy quantifiers, Q_3 is fuzzy quantifier which is to be determined, and *A, B, C, D, E* and *F* are interrelated fuzzy predicates.

In what follows, we shall present a brief discussion of two basic types of fuzzy syllogisms. A more detailed description of these and other fuzzy syllogisms may be found in Zadeh (1983c, 1984).

The ***intersection/product syllogism*** may be viewed as an instance of (5.1) in which

3. For consistency with the definition of A, S must be such that if u and u' have a high degree of similarity, then their grades of membership in A should be close in magnitude.

$$C \triangleq A \text{ and } B$$
$$E \triangleq A$$
$$F \triangleq B \text{ and } D \ ,$$

and $Q_3 = Q_1 \otimes Q_2$, i.e., Q_3 is the product of Q_1 and Q_2 in fuzzy arithmetic. Thus, we have as the statement of the syllogism:

$$\frac{\begin{array}{l} Q_1\ A's \text{ are } B's \\ Q_2(A \text{ and } B)'s \text{ are } C's \end{array}}{(Q_1 \otimes Q_2)\ A's \text{ are } (B \text{ and } C)'s\ .} \qquad (5.2)$$

In particular, if B is contained in A, i.e., $\mu_B \leq \mu_A$, where μ_A and μ_B are the membership functions of A *and* B, respectively, then A *and* $B = B$, and (5.2) becomes

$$\frac{\begin{array}{l} Q_1\ A's \text{ are } B's \\ Q_2\ B's \text{ are } C's \end{array}}{(Q_1 \otimes Q_2)\ A's \text{ are } (B \text{ and } C)'s\ .} \qquad (5.3)$$

Since B *and* C implies C, it follows at once from (5.3) that

$$\frac{\begin{array}{l} Q_1\ A's \text{ are } B's \\ Q_2\ B's \text{ are } C's \end{array}}{\geq (Q_1 \otimes Q_2)\ A's \text{ are } C's\ ,} \qquad (5.4)$$

which is the *chaining syllogism* expressed by (1.4). Furthermore, if the quantifiers Q_1 and Q_2 are monotonic, i.e., $\geq Q_1 = Q_1$ and $\geq Q_2 = Q_2$, then (5.4) becomes the *product syllogism*

$$\frac{\begin{array}{l} Q_1\ A's \text{ are } B's \\ Q_2\ B's \text{ are } C's \end{array}}{(Q_1 \otimes Q_2)\ A's \text{ are } C's} \qquad (5.5)$$

In the case of the *consequent conjunction syllogism*, we have

$$C \triangleq A$$
$$E \triangleq A$$
$$F = B \text{ and } D\ .$$

In this case, the statement of syllogism is:

$$\frac{\begin{array}{l} Q_1\ A's \text{ are } B's \\ Q_2\ A's \text{ are } C's \end{array}}{Q_3\ A's \text{ are } (B \text{ and } C)'s} \qquad (5.6)$$

where Q is a fuzzy number (or interval) defined by the inequalities

$$0 \otimes (Q_1 \oplus Q_2 \ominus 1) \leq Q \leq Q_1 \otimes Q_2\ , \qquad (5.7)$$

where $\oplus$, $\ominus$, $\otimes$ and $\oslash$ are the operations of addition, subtraction, min and max in fuzzy arithmetic.

As a simple illustration, consider the dispositions

$$d_1 \triangleq \textit{students are young}$$
$$d_2 \triangleq \textit{students are single}\ .$$

Upon restoration, these dispositions become the propositions

$$p_1 \triangleq \textit{most students are young}$$
$$p_2 \triangleq \textit{most students are single}$$

Then, applying the consequent conjunction syllogism to p_1 and p_2, we can infer that

Q students are single and *young*

where

$$2\ most \ominus 1 \leq Q \leq most\ . \qquad (5.8)$$

Thus, from the dispositions in question we can infer the disposition

$$d \triangleq \textit{students are single} \text{ and } \textit{young}$$

on the understanding that the implicit fuzzy quantifier in d is expressed by (5.8).

6. Negation of Dispositions

In dealing with dispositions, it is natural to raise the question: What happens when a disposition is acted upon with an operator, T, where T might be the operation of negation, active-to-passive transformation, etc. More generally, the same question may be asked when T is an operator which is defined on pairs or n-tuples of dispositions.

As an illustration, we shall focus our attention on the operation of negation. More specifically, the question which we shall consider briefly is the following: Given a disposition, d, what can be said about the negaton of d, *not d*? For example, what can be said about *not (birds can fly)* or *not (young men like young women)*.

For simplicity, assume that, after restoration, d may be expressed in the form

$$rd \triangleq Q\ A's \text{ are } B's\ . \qquad (6.1)$$

Then,

$$not\ d = not\ (Q\ A's \text{ are } B's)\ . \qquad (6.2)$$

Now, using the semantic equivalence established in Zadeh (1978), we may write

$$not\ (Q\ A's \text{ are } B's) \equiv (not\ Q)\ A's \text{ are } B's\ , \qquad (6.3)$$

where *not* Q is the complement of the fuzzy quantifier Q in the sense that the membership function of *not* Q is given by

$$\mu_{not\ Q}(u) = 1\text{-}\mu_Q(u)\ , 0 \leq u \leq 1\ . \qquad (6.4)$$

Furthermore, the following inference rule can readily be established (Zadeh, 1983a):

$$\frac{Q\ A's \text{ are } B's}{\geq (ant\ Q)\ A's \text{ are not } B's}\ , \qquad (6.5)$$

where *ant* Q denotes the *antonym* of Q, defined by

$$\mu_{antQ}(u) = \mu_Q(1\text{-}u)\ ,\ 0 \leq u \leq 1\ , \qquad (6.6)$$

On combining (6.3) and (6.5), we are led to the following result:

$$not(Q\ A's \text{ are } B's) = \geq (ant\ (not\ Q))\ A's \text{ are not } B's \qquad (6.7)$$

which reduces to

$$not(Q\ A's \text{ are } B's) = (ant\ (not\ Q))\ A's \text{ are not } B's \qquad (6.8)$$

if Q is monotonic (e.g., $Q \triangleq most$).

As an illustration, if $d \triangleq$ *birds can fly* and $Q \triangleq most$, then (6.8) yields

$$not\ (birds\ can\ fly)\ (ant\ (not\ most))\ birds\ cannot\ fly\ . \qquad (6.9)$$

It should be observed that if Q is an approximation to *all*, then *ant(not Q)* is an approximation to *some*. For the right-hand member of (6.9) to be a disposition, *most* must be an approximation to *at least a half.* In this case *ant (not most)* will be an approximation to *most,* and consequently the right-hand member of (6.9) may be expressed -- upon the suppression of *most* -- as the disposition *birds cannot fly.*

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