

# Language Models

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- Role of LM in ASR and SMT
- N-gram Language Models
- Evaluation of Language Models
- Frequency Smoothing
- Frequency Discounting
- Is smoothing necessary?
- ARPA LM representation
- New IRSTLM Toolkit

**Goal:** find the words  $\mathbf{w}^*$  in a speech signal  $\mathbf{x}$  such that:

$$\mathbf{w}^* = \arg \max_{\mathbf{w}} \Pr(\mathbf{x} | \mathbf{w}) \Pr(\mathbf{w}) \quad (1)$$

**Problems:**

- **language modeling (LM):** estimating  $\Pr(\mathbf{w})$
- **acoustic modeling (AM):** estimating  $\Pr(\mathbf{x} | \mathbf{w})$
- **search problem:** computing (1)

AM sums over hidden state sequences  $\mathbf{s}$  a Markov process of  $(\mathbf{x}, \mathbf{s})$  from  $\mathbf{w}$

$$\Pr(\mathbf{x} | \mathbf{w}) = \sum_{\mathbf{s}} \Pr(\mathbf{x}, \mathbf{s} | \mathbf{w})$$

**Hidden Markov Model:** hidden states "link" speech frames to words.

**Goal:** find the English string  $\mathbf{f}$  translating the foreign text  $\mathbf{e}$  such that:

$$\mathbf{e}^* = \arg \max_{\mathbf{e}} \Pr(\mathbf{f} | \mathbf{e}) \Pr(\mathbf{e}) \quad (2)$$

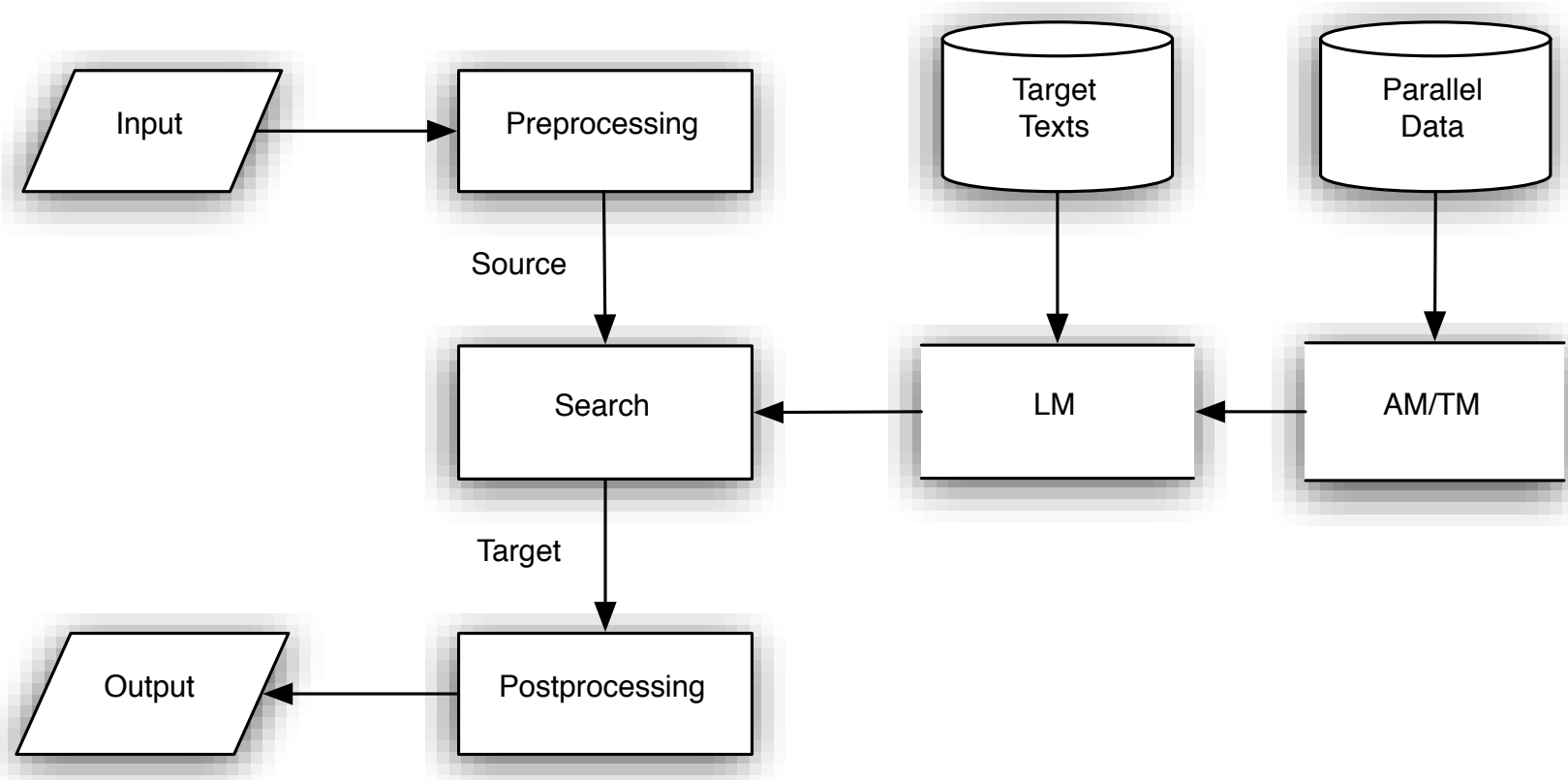
**Problems:**

- **language modeling** (LM): estimating  $\Pr(\mathbf{e})$
- **translation modeling** (TM): estimating  $\Pr(\mathbf{f} | \mathbf{e})$
- **search** problem: computing (2)

TM sums over hidden alignments  $\mathbf{a}$  a stochastic process generating  $(\mathbf{f}, \mathbf{a})$  from  $\mathbf{e}$ .

$$\Pr(\mathbf{f} | \mathbf{e}) = \sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} | \mathbf{e})$$

**Alignment Models:** hidden alignments "link" foreign words with English words.



- **Parallel data** are samples of observations  $(\mathbf{x}, \mathbf{w})$  and  $(\mathbf{f}, \mathbf{e})$
- AM and TM can be **machine-learned** without observing  $\mathbf{s}$  and  $\mathbf{a}$ .

- Translation hypotheses are ranked by:

$$\mathbf{e}^* = \operatorname{argmax}_e \sum_a \sum_k \lambda_k \log h_k(\mathbf{e}, \mathbf{f}, \mathbf{a})$$

- **Phrases** are finite string (cf. n-grams)
- Hidden variable  $\mathbf{a}$  embeds:
  - **segmentation** of  $\mathbf{f}$  and  $\mathbf{e}$  into phrases
  - **alignment** of phrases of  $\mathbf{f}$  with phrases of  $\mathbf{e}$
- **Feature functions**  $h_k()$  include:
  - Translation Model: appropriateness of phrase-pairs
  - Distortion Model: word re-ordering
  - **Language Model**: fluency of target string
  - Length Model: number of target words
- Role of the LM is exactly the same as for the classical approach:
  - to score translations generated incrementally by the search algorithm!

**Goal:** given a text  $w_1^T = w_1 \dots, w_t, \dots, w_T$  we can compute its probability by:

$$\Pr(w_1^T) = \Pr(w_1) \prod_{t=2}^T \Pr(w_t | h_t) \quad (3)$$

where  $h_t = w_1, \dots, w_{t-1}$  indicates the **history** of word  $w_t$ .

- $\Pr(w_t | h_t)$  becomes difficult to estimate as the history  $h_t$  grows .
- hence, we take the  $n$ -gram **approximation**  $h_t \approx w_{t-n+1} \dots w_{t-1}$

e.g. Full history:  $\Pr(\text{Parliament} | \text{I declare resumed the session of the European})$

**3-gram** :  $\Pr(\text{Parliament} | \text{the European})$

The choice of  $n$  determines the complexity of the LM (# of parameters):

- **bad**: no magic recipe about the optimal order  $n$  for a given task
- **good**: language models can be evaluated quite cheaply

- Indirect: **impact on task** (e.g. BLEU score for MT)
- Direct: capability of **predicting words**

The **perplexity** (PP) measure is defined as: <sup>1</sup>

$$PP = 2^{LP} \quad \text{where} \quad LP = -\frac{1}{M} \log_2 p(w_1^M) \quad (4)$$

- $w_1^M$  is a **sufficiently long test sample** and  $p(w_1^M)$  is the LM probability

**Properties:**

- $0 \leq PP \leq |V|$  (size of the vocabulary  $V$ )
- **predictions** are as good as guessing among  $PP$  equally likely options

**Good:** there is typical strong correlation between PP and BLUE scores!

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<sup>1</sup>[Exercise 1. Find PP of 1-gram LM  $p(T)=0.5$  on T H T H T H T T H T T H. ]



Estimating  $n$ -gram probabilities is not trivial due to:

- **parameter space**: with 10,000-word  $V$  we can form one trillion 3-grams!
- **data sparseness**: most of 3-grams are rare events even in large corpora.

**Relative frequency estimate**: MLE of any discrete conditional distribution is:

$$f(w \mid x \ y) = \frac{c(x \ y \ w)}{\sum_w c(x \ y \ w)}$$

where counts  $c(\cdot)$  are taken over a **large training corpus**.

**Problem**: relative frequencies in general over-fit the training data

- if the test sample contains a "new"  $n$ -gram **PP**  $\rightarrow +\infty$
- with 4-grams or 5-grams LM this is largely the most frequent case!

**We need frequency smoothing!**

**Issue:**  $f(w | x y) > 0$  only if  $w$  was observed after  $x y$  in the training data.

**Idea:** for each  $w$  take off some fraction of probability from  $f(w | x y)$  and redistribute the total to words never observed after  $x y$ .

- the discounted frequency  $f^*(w | x y)$  satisfies:

$$0 \leq f^*(w | x y) \leq f(w | x y) \quad \forall x, y, w \in V$$

Notice: in general  $f^*(w | x y)$  does not sum up to 1!

- the "total discount" is called zero-frequency probability  $\lambda(x y)$ <sup>2</sup>:

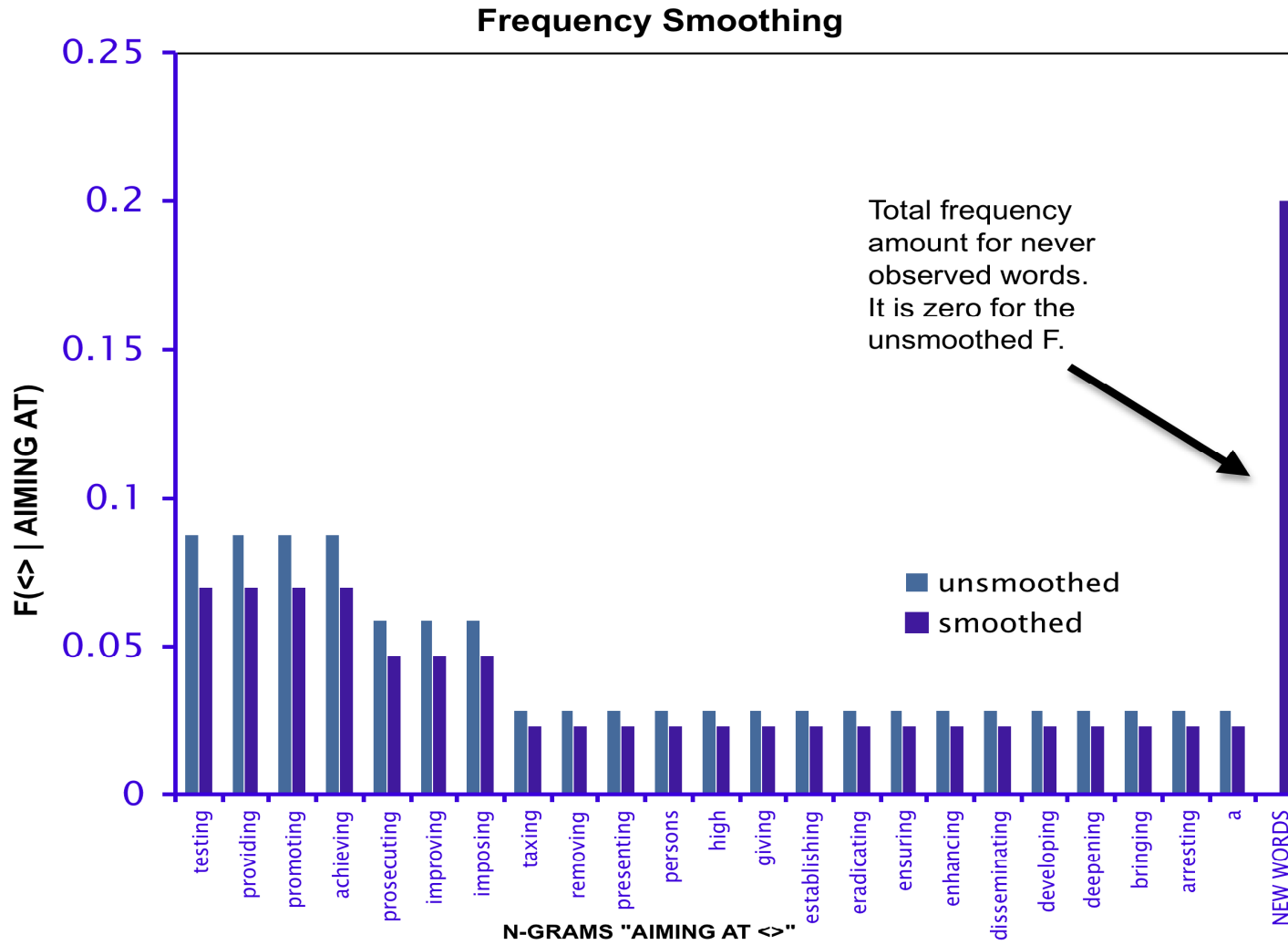
$$\lambda(x y) = 1.0 - \sum_{w \in V} f^*(w | x y)$$

How to redistribute the total discount?

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<sup>2</sup>Notice: by convention  $\lambda(x y) = 1$  if  $f(w | x y) = 0$  for all  $w$ , i.e.  $c(x y) = 0$ .

# Discounting Example



**Insight:** redistribute  $\lambda(x \ y)$  according to the lower-order smoothed frequency.

Two major **hierarchical** schemes to compute the **smoothed frequency**  $p(w \mid x \ y)$ :

- **Back-off**, i.e. select the best available  $n$ -gram approximation:

$$p(w \mid x \ y) = \begin{cases} f^*(w \mid x \ y) & \text{if } f^*(w \mid x \ y) > 0 \\ \alpha_{xy} \lambda(x \ y) p(w \mid y) & \text{otherwise} \end{cases} \quad (5)$$

where  $\alpha_{xy}$  is an appropriate normalization term.<sup>3</sup>

- **Interpolation**, i.e. sum up the two approximations:

$$p(w \mid x \ y) = f^*(w \mid x \ y) + \lambda(x \ y) p(w \mid y). \quad (6)$$

Smoothed frequencies are learned bottom-up, starting from 1-grams ...

<sup>3</sup>[Exercise 2. Find an expression for  $\alpha_{xy}$  s.t.  $\sum_w p(w \mid x \ y) = 1$ .]

Unigram smoothing permits to treat **out-of-vocabulary** (OOV) words in the LM.

**Assumptions:**

- $|U|$  is an upper-bound estimate of the size of language vocabulary
- $f^*(w)$  is strictly positive on the observed vocabulary  $V$
- $\lambda$  is the total discount reserved to OOV words

**Then:** 1-gram back-off and interpolation collapse to:

$$p(w) = \begin{cases} f^*(w) & \text{if } w \in V \\ \lambda(|U| - |V|)^{-1} & \text{otherwise} \end{cases} \quad (7)$$

**Notice:** LMs make also other approximations when an OOV word  $x$  appears:

$$p(w \mid h_1 x h_1) = p(w \mid h_2) \quad \text{and} \quad p(x \mid h) = p(x)$$

**Important:** use a common value  $|U|$  when comparing/combining different LMs!

## Witten-Bell estimate (WB) [Witten and Bell, 1991]

- **Insight:** learn  $\lambda(x y)$  by counting "new word" events in 3-grams  $x y *$ 
  - corpus:  $x y \mathbf{u} x x y \mathbf{t} t x y \mathbf{u} w x y \mathbf{w} x y \mathbf{t} u x y \mathbf{u} x y \mathbf{t}$
  - then  $\lambda(x y) \propto$  number of "new word" events (i.e. 3)
  - and  $f^*(w | x y) \propto$  relative frequency (linear discounting)
- **Solution:**

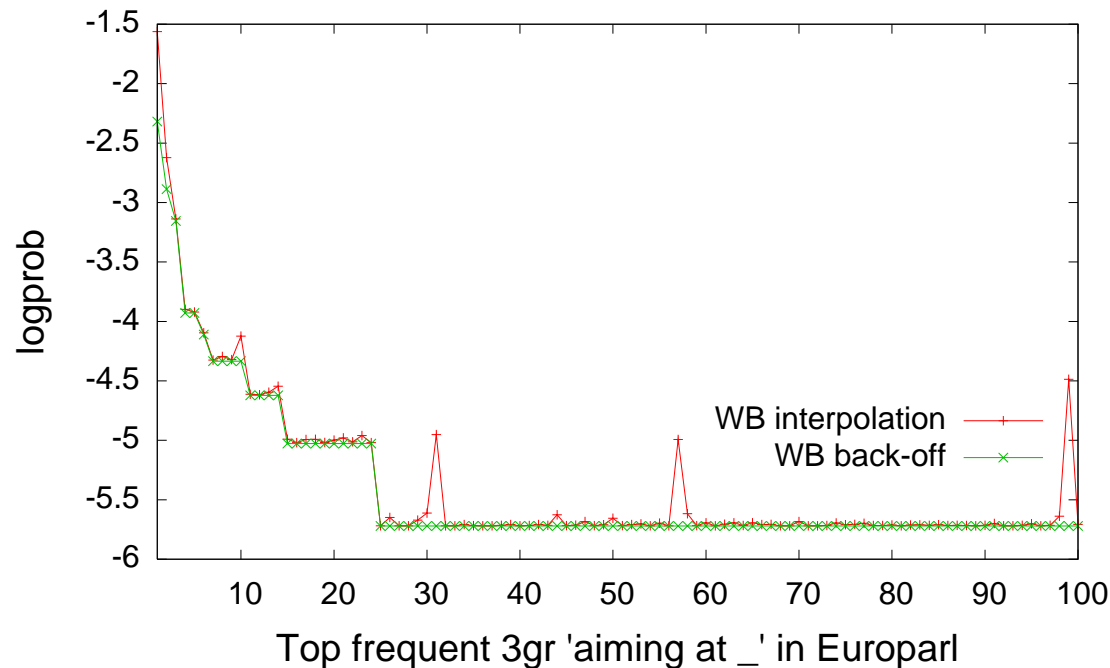
$$\lambda(x y) = \frac{n(x y *)}{c(x y) + n(x y *)} \quad \text{and} \quad f^*(w | xy) = \frac{c(x y w)}{c(x y) + n(x y *)}$$

where  $c(x y) = \sum_w c(x y w)$  and  $n(x y *) = |\{w : c(x y w) > 0\}|$ .<sup>4</sup>

- **Pros:** easy to compute, robust for small corpora, works with artificial data.
- **Cons:** underestimates probability of frequent  $n$ -grams

<sup>4</sup>[Exercise 3. Compute  $f^*(u | x y)$  with WB on the above artificial text.]

- interpolation and back-off with WB discounting
- trigram LMs estimated on the English Europarl corpus
- logprobs of 3-grams of type aiming at \_ observed in training



- peaks correspond to very probable 2-grams interpolated with  $f^*$  respectively: at that, at national, at European
- Practically, [interpolation and back-off perform similarly](#)

## Absolute Discounting (AD) [Ney and Essen, 1991]

- **Insight:**
  - discount by subtracting a small constant  $\beta$  ( $0 < \beta \leq 1$ ) from each counts
  - estimate  $\beta$  by maximizing the leaving-one-out likelihood of the training data
- **Solution:** (notice: one distinct  $\beta$  for each  $n$ -gram order)

$$f^*(w | x y) = \max \left\{ \frac{c(xyw) - \beta}{c(xy)}, 0 \right\} \text{ which gives } \lambda(xy) = \beta \frac{\sum_{w:c(xyw)>1} 1}{c(xy)}$$

where  $\beta \approx \frac{n_1}{n_1 + 2n_2} < 1$  and  $n_r = |\{x y w : c(x y w) = r\}|$ .<sup>5</sup>

- **Pros:** easy to compute, accurate estimate of frequent  $n$ -grams.
- **Cons:** problematic with small and artificial samples.

<sup>5</sup>[Exercise 4. Given the text in WB slide find the number of 3-grams,  $n_1$ ,  $n_2$ ,  $\beta$ ,  $f^*(w | x y)$  and  $\lambda(x y)$ ]



## Kneser-Ney method (KN) [Kneser and Ney, 1995]

- **Insight:** 2-grams counts should correspond to the "back-off" cases
  - count all "back-off" events in 3-grams of type  $* y w$  (cf. WB method)
  - corpus: **x y w x t y w t x y w u y w t y w u x y w u u y w**
  - corrected counts  $n(* y w) =$  number of observed back-offs (i.e. 3)
- **Solution:** (for 3-gram normal counts)

$$f^*(w | y) = \max \left\{ \frac{n(* y w) - \beta}{n(* y *)}, 0 \right\} \text{ which gives } \lambda(y) = \beta \frac{\sum_{w: n(* y w) > 1} 1}{n(* y *)}$$

where  $n(* y w) = |\{x : c(x y w) > 0\}|$  and  $n(* y *) = |\{x w : c(x y w) > 0\}|$

- **Pros:** better back-off probabilities, can be applied to other methods
- **Cons:** LM cannot be used to compute lower order  $n$ -gram probs

## Modified Kneser-Ney (MKN) [Chen and Goodman, 1999]

- **Insight:** specific discounting coefficients for unfrequent  $n$ -grams
- **Solution:**

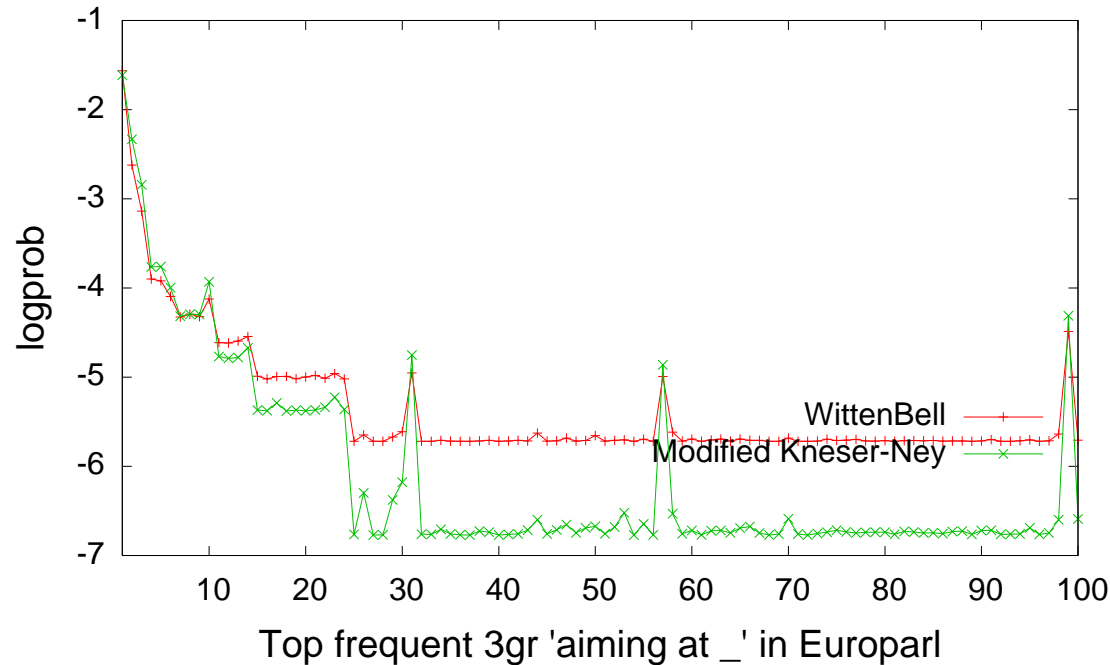
$$f^*(w | x y) = \frac{c(x y w) - \beta(c(x y w))}{c(x y)}$$

where  $\beta(0) = 0$ ,  $\beta(1) = D_1$ ,  $\beta(2) = D_2$ ,  $\beta(c) = D_{3+}$  if  $c \geq 3$ , coefficients are computed from  $n_r$  statistics, corrected counts used for lower order  $n$ -grams

- **Pros:** see previous + more fine grained smoothing
- **Cons:** see previous + more sensitiveness to noise

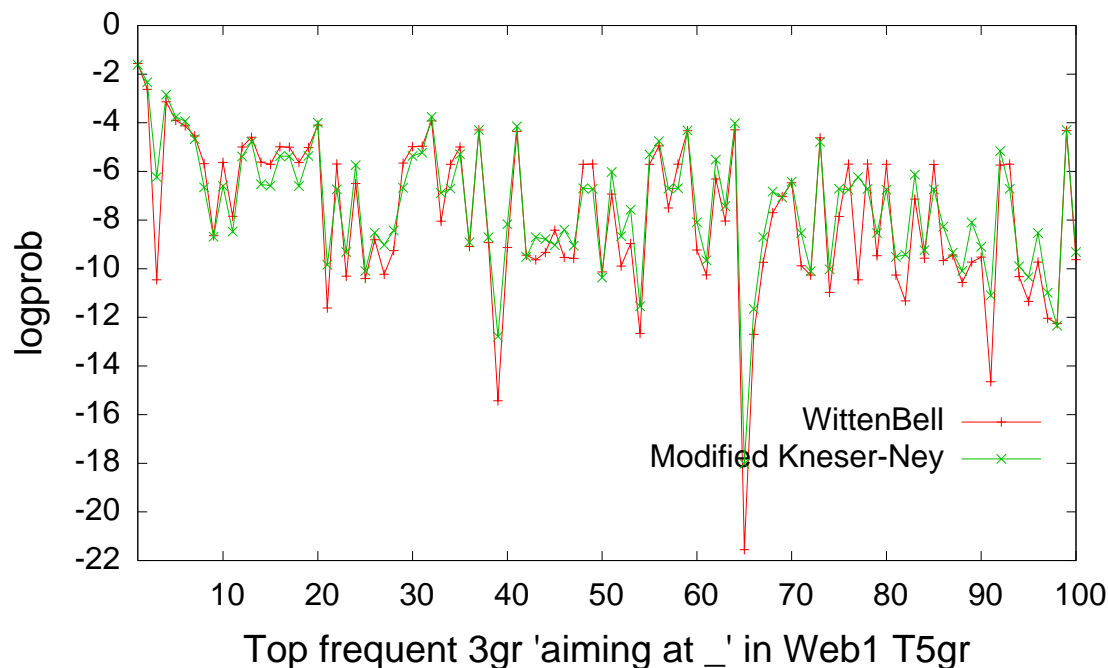
**Important:** LM interpolation with MKN is the most popular training method. Under proper training conditions it gives the best PP and BLEU scores!

- interpolation with WB and MKN discounting trained on Europarl
- the plot shows the logprob of observed 3-grams of type aiming at \_



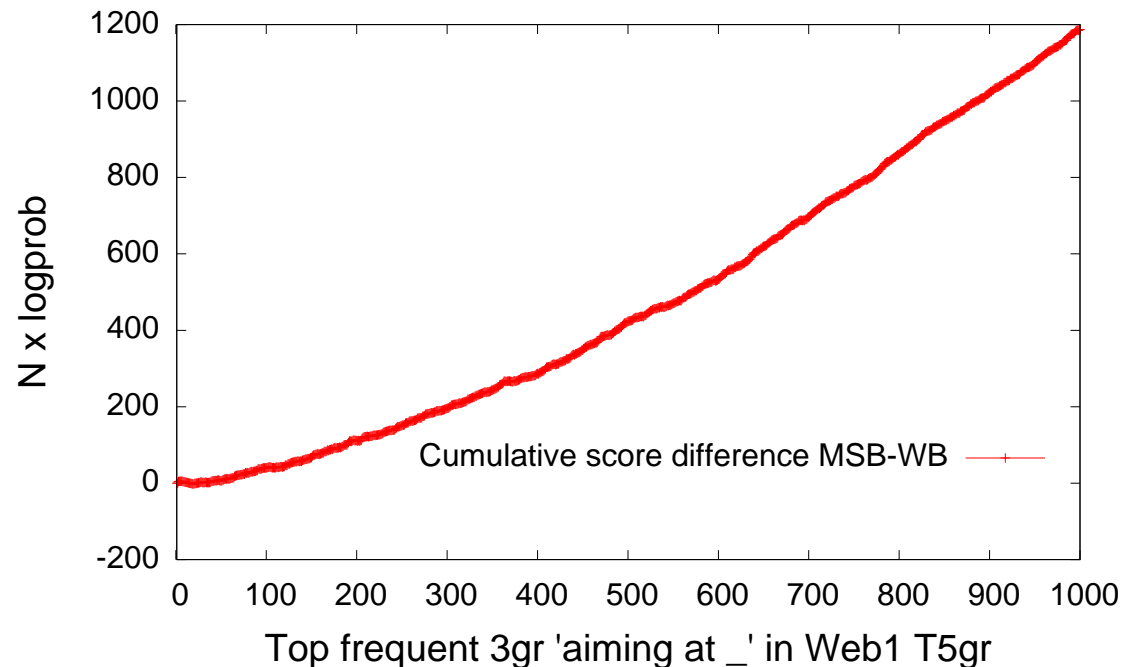
- notice that for less frequent 3-grams WB assigns higher probability
- we have three very high peaks corresponding large corrected counts:  
 $n(*at\ that)=665$   $n(*\ at\ national)=598$   $n(*\ at\ European)=1118$
- also an interesting peak at rank #26:  $n(*\ at\ very)=61$

- train: interpolation with WB and MKN discounting on Europarl
- test: 3-grams of type aiming at \_ are from the Google 1TWeb sample



- the trend is the same but MKN outperforms WB smoothing
- If you don't believe, check the next slide ....

- train: interpolation with WB and MKN discounting on Europarl
- test: 3-grams of type aiming at \_ are from the Google 1TWeb sample
- plot: cumulative score differences between MKN and WB on top 1000 3-grams

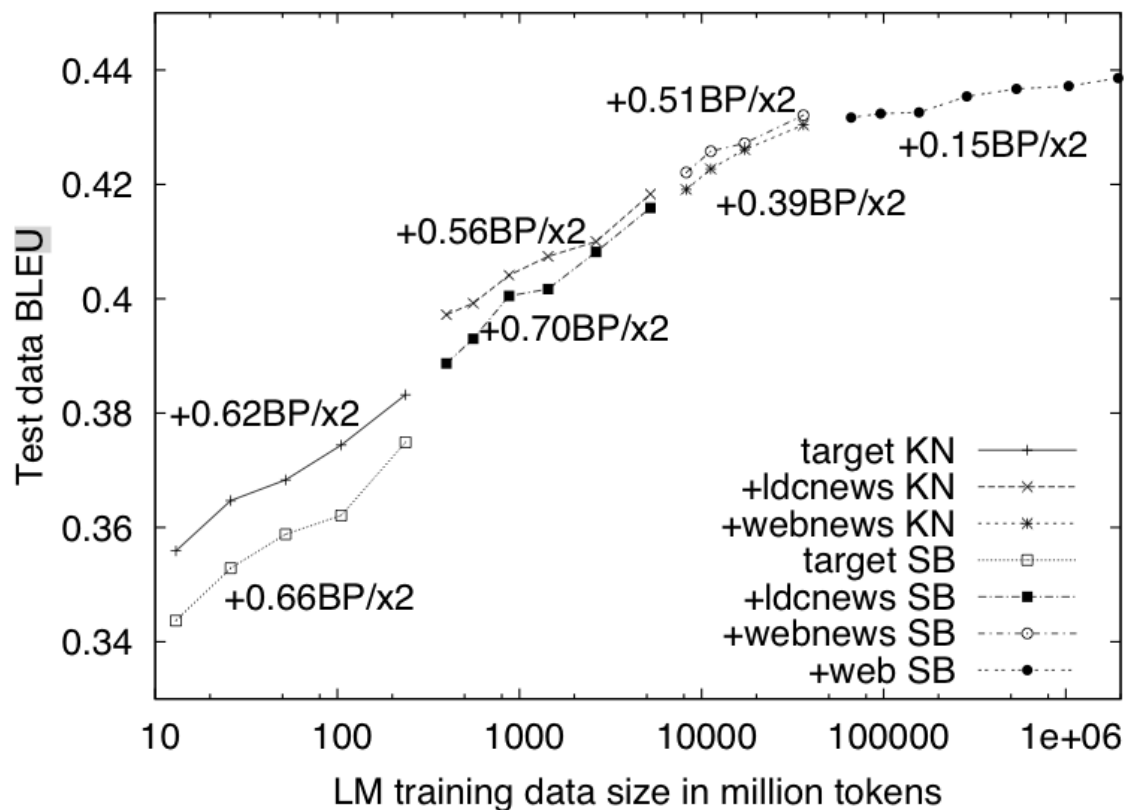


- **LM Quantization** [Federico and Bertoldi, 2006]
  - **Idea**: one codebook for each n-gram/back-off level
  - **Pros**: improves storage efficiency
  - **Cons**: reduces discriminatory power
  - Experiments with 8bit quantization on ZH-EN NIST task showed:
    - \* 2.7% BLUE drop with a 5-gram LM trained on 100M-words
    - \* 1.6% BLUE drop with a 5-gram LM trained on 1.7G words.
- **Stupid back-off** [Brants et al., 2007]
  - simple smoothing, no correct normalization

$$p(w | x y) = \begin{cases} f(w | x y) & \text{if } f(w | x y) > 0 \\ k \cdot p(w | y) & \text{otherwise} \end{cases} \quad (8)$$

where  $k = 0.4$  and  $p(w) = c(w)/N$ .

## Stupid back-off (SB) versus Modified Kneser-Ney (KN)



From [Brants et al., 2007].

- Conclusion: proper smoothing useful up to 1 billion word training data?

Represents both interpolated and back-off n-gram LMs

- format:  $\log(\text{smoothed-freq}) :: \text{n-gram} :: \log(\text{back-off weight})$
- computation: look first for smoothed-freq, otherwise back-off

```
\data\  
ngram 1= 86700  
ngram 2= 1948935  
ngram 3= 2070512  
\1-grams:  
-2.88382      !          -2.38764  
-2.94351      world      -0.514311  
-6.09691      dublin      -0.15553  
...  
\2-grams:  
-3.91009      world !      -0.351469  
-3.91257      hello world -0.24  
-3.87582      hello dublin -0.0312  
..  
\3-grams:  
-0.00108858   hello world !  
-0.000271867   , hi hello !  
...  
\end\  

```

$\log\text{Pr}(! | \text{hello dublin}) = -0.0312 + \log\text{Pr}(! | \text{dublin})$

$\log\text{Pr}(! | \text{dublin}) = -0.15553 - 2.88382$



## Main Features [Federico et al., 2008]

- **Single thread training for standard LMs**
  - all major smoothing methods: WB, AD, MKN, ...
  - LM pruning, internal/external interpolation, adaptation
- **Distributed training for huge LMs**
  - simple smoothing methods: interpolation with WB, KN
  - split dictionary into balanced  $n$ -gram prefix lists
  - collect  $n$ -grams for each prefix lists
  - estimate and merge single LMs for each prefix list
- **Space optimization**
  - $n$ -gram collection uses dynamic storage to encode counters
  - distributed LM estimation just requires reading disk files
  - probs and back-off weights are quantized
  - run-time LM data structure is loaded on demand
- **LM caching**
  - computations of probs, access to internal lists, LM states, ....

## References

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