Smoothed Bloom Filter Language Models

Saving Space by Flipping Coins

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First Machine Translation Marathon (16th April 2007)

Outline

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1. **Motivation**
   - Scaling Language Modelling
   - Problems with Lossless Representations
   - Lossy Representations
2. **Bloom Filter Language Models**
   - The Bloom Filter
   - Extending the BF for Language Modelling
3. Experiments

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Language Modelling Challenges

- Good modelling
  - N-grams (n = 8?)
- Good estimation
  - Millions / billions / trillions of words
  - Good estimators (e.g., Witten-Bell, Kneser-Ney)
- Small memory footprint
- Low computational complexity

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A Curse of Dimensionality - and Large Corpora

- Size of N-gram event space increases exponentially

$$|\mathcal{U}_N| = |vocab|^N$$

- Set of observed N-grams n increases more slowly

$$n \ll |vocab| \times 50^{N-1}$$

- These are *very* different quantities

Some Corpus Statistics

Corpus	Gigaword	Europarl	GW Apriori	EP Apriori
1-gms	281K	61K	281K	61K
2-gms	5,441K	127K	78,961,000K	3,721,000K
3-gms	274,844K	467K	etc.	
4-gms	599,383K	815K		
5-gms	842,297K	1,028K		

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Information-based Space Lower Bound

Statement

$log_2 \binom{|\mathcal{U}|}{n}$ bits are needed to represent n items from a Universe $\mathcal{U}$

Why

- There are $\binom{|\mathcal{U}|}{n}$ distinct sets of size n in $\mathcal{U}$
- A distinct code must be assigned to each such set
- $log_2(x)$ bits are needed to represent x distinct codes

Problem

Any lossless representation scales with $|\mathcal{U}|$ (this is not good)

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All Language Models are Approximate

- Model assumptions are *approximate*
- Not using all available data is *approximate*
- Model reduction - pruning, clustering etc. - is *approximate*
- Parameter estimates are *approximate*

Bloom filters

Are also approximate but may reduce the above approximations

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Representing a Set via Hashing

Problem

Represent a set $\mathcal{S}$ of size n drawn from $\mathcal{U}$ where $n \ll |\mathcal{U}|$

Solution

Bloom Filter uses a bitarray of size m and k hash functions

To Train:

- Hash each item k times setting corresponding bits in m

To Test:

- Hash a candidate k times, if all bits set report *member* else *non-member*

Representing a Set via Hashing

Bloom filter (cont.)

- False positives occur with quantifiable probability
- Size and false positive rate *independent* of $|\mathcal{U}|$ (in theory)
- No false negative - i.e., *one-sided error*

E.g. 7.2 bits per item $\rightarrow$ false positive rate ≈ 0.03

Using a Bloom Filter

0	0	0	0	0	0	0	0	0	0	0	0	0

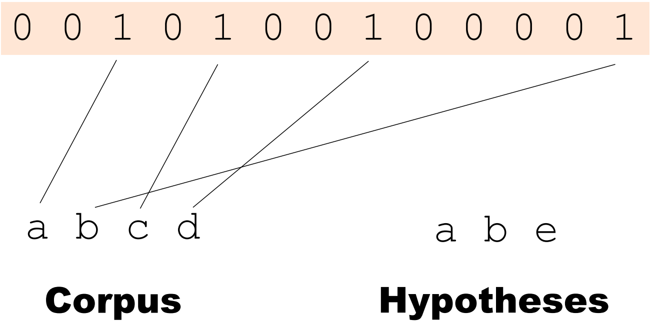
a b c d

Corpus

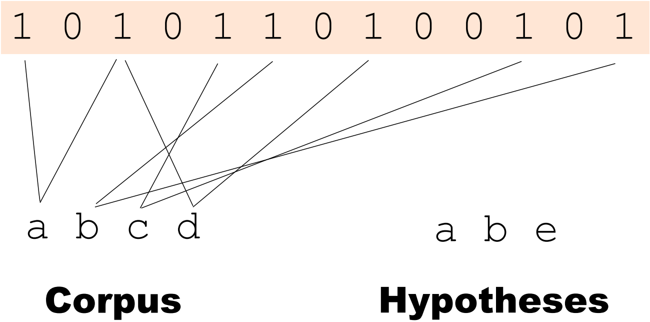
a b e

Hypotheses

Using a Bloom Filter



Using a Bloom Filter



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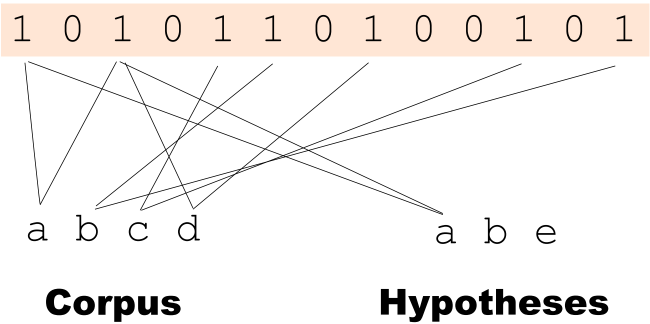

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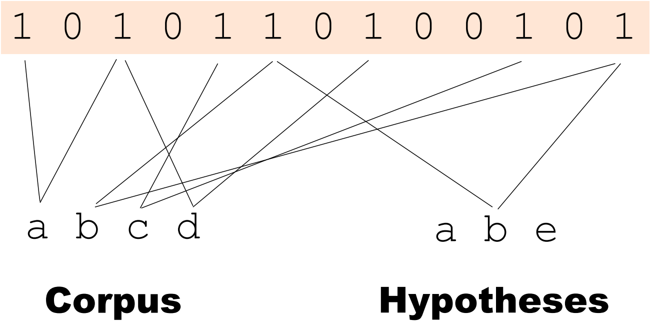

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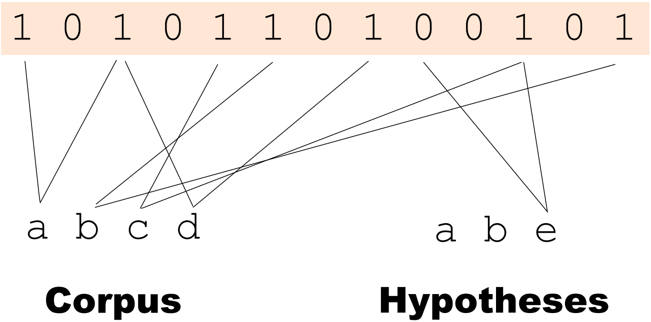

Optimising a Bloom Filter

How many hash functions?

False positive probability: $f = (1 - p)^k$
where $p = (1 - \frac{1}{m})^{kn}$ is the probability that a bit is still zero
f is minimized for: $k^* = \frac{m}{n}\ln(2)$

Previous Example: $m = 13, n = 4$

With $k = 1$ the false positive rate was $\frac{4}{13} \approx 0.30$
With $k = 2$ the false positive rate was $(\frac{7}{13})^2 \approx 0.28$

Asymptotically setting half the bits is optimal

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Storing Corpus Statistics

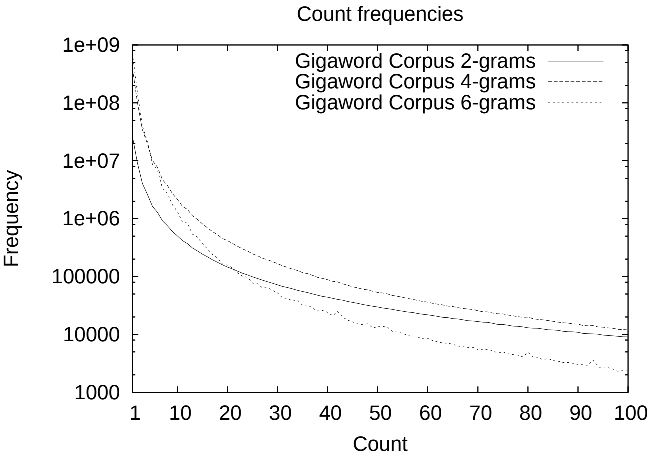
Problem

Bloom filters are *not* an associative data structure

Possible Solutions

1. Append each N-gram in set by its count
   - False positive rate will increase by factor $|MAXCOUNT|$
   - Error will no longer be *one-sided*
2. Replace each bit by a counter
   - Space increased by factor $\log(|MAXCOUNT|)$
   - Most counters will be set to 1 or 2

Storing Corpus Statistics



Storing Corpus Statistics

Our Solution

Store each N-gram $1 + \lfloor log(count) \rfloor$ times

Log Frequency Bloom filter

- Store each N-gram appended by an integer j

$$1 \geq j \geq 1 + \lfloor log(count) \rfloor$$

- Query an N-gram's frequency by appending an integer $j = 1$ and incrementing until hitting a 0

Estimation errors decay exponentially: $f(d) = f^d$ for $d > 0$

Converting Corpus Frequencies to a Set

Raw Counts		Quant Counts
the cat	15	4
the hat	3	2
the mat	1	1
the eggs	1	1
the bacon	1	1

Transformed Set

{the cat_1, the cat_2, the cat_3, the cat_4, the hat_1, the hat_2, the mat_1, the eggs_1, the bacon_1}

Storing Related Events

Language Model Statistics

- Witten-Bell: N-gram and suffix counts
- Kneser-Ney: N-gram, prefix, suffix and infix counts

Proxy Events

- Use existence of one event to *infer* a related event
  e.g. presence of $N - 1$-gram implies suffix count ≥ 1

Savings for Witten-Bell

- No need to store singleton suffix counts
- Reduced set $\approx \frac{2}{3}$ size of complete set

Reducing Effective Error Rate

Actual Error Rate

Errors only occur for non-members (i.e. one-sided error)

$$err = Pr(x \notin Corpus | x \in Hypothesis) \times f$$

Can we increase the *a priori* membership probability?

Using Monotonicity of *N*-gram Event Space

- If a unigram x tests false, then a bigram xy *cannot* be a member
- More generally, $freq(xy) \leq \min\{freq(x), freq(y)\}$

An Example

Interpolated Witten-Bell BF-LM

$$P_{wb}(w_i|w_{i-n+1}^{i-1}) = \lambda_{w_{i-n+1}^{i-1}} P_{ml}(w_i|w_{i-n+1}^{i-1}) + (1-\lambda_{w_{i-n+1}^{i-1}}) P_{wb}(w_i|w_{i-n+2}^{i-1})$$

where λ_x is defined via,

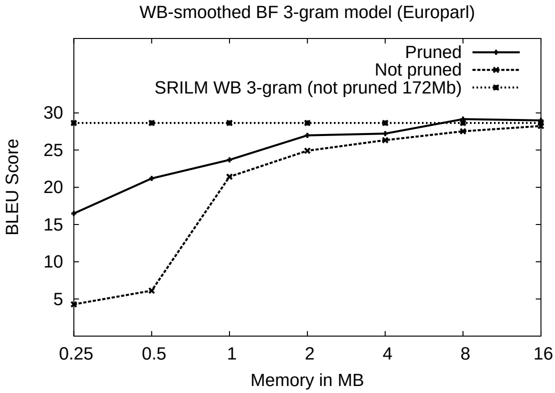
$$1 - \lambda_x = \frac{count(x)}{suffix(x) + count(x)},$$

- Start from lowest order event (i.e. unigram)
- Bound numerator in ml term by count of denominator
- Bound suffix count by its token frequency
- Truncate computation if ml denominator is zero

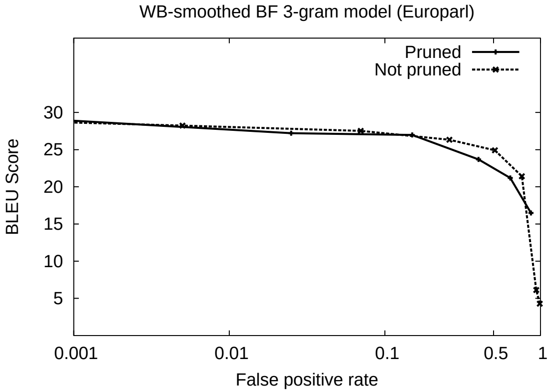
Baseline Models Europarl Witten-Bell

n	Pruned	Types	Mem.	Gzip'd	BLEU
3	No	5.9M	172Mb	51Mb	28.95
3	Yes	2.4M	64Mb	21Mb	28.96
4	No	14.1M	477Mb	129Mb	28.99
4	Yes	3.5M	102Mb	33Mb	29.41
5	No	24.2M	924Mb	238Mb	29.38
5	Yes	4.2M	131Mb	38Mb	29.60

Witten-Bell BF 3-gram Europarl



Witten-Bell BF 3-gram Europarl



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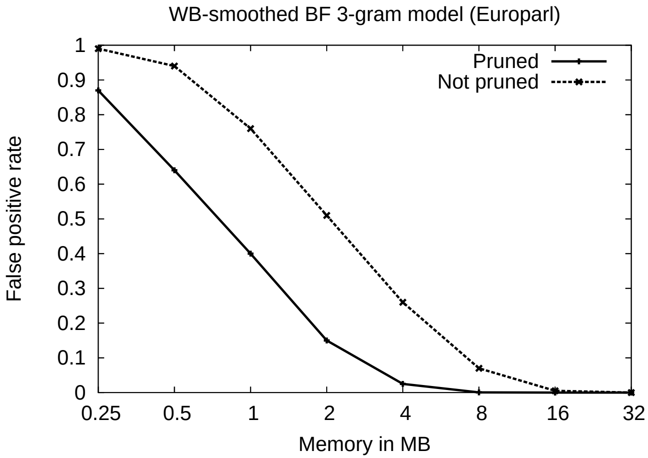

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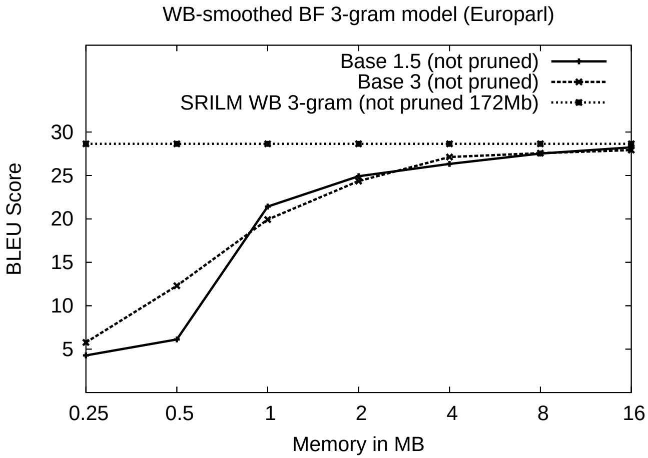
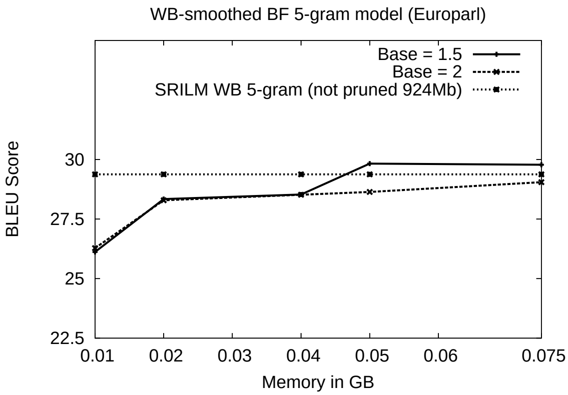
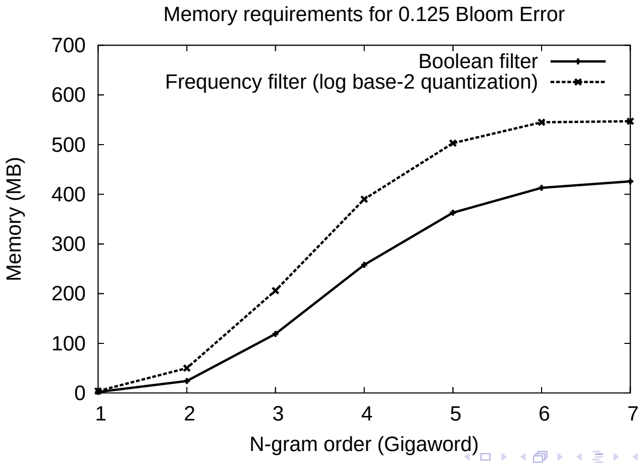


Witten-Bell BF 5-gram Europarl

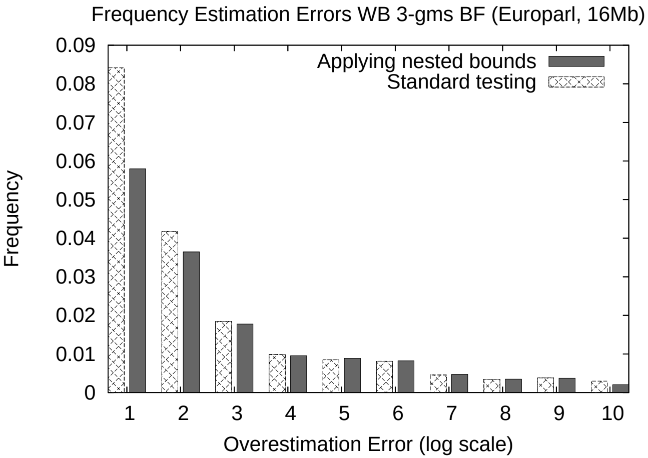


Log Frequency Scheme for Corpus Statistics

- Set increases by *less than 2* when storing frequencies



Applying Nested Bounds



Summary

- Bloom filters can be used effectively for language modelling below information-theoretic lower bounds
- 11 - 15 bits per *N*-gram seems like enough

- Future Work
  - Reducing computation in the log frequency BF scheme
  - Hybrid models - e.g. explicit 1,2-grams + BF 3,4,5-grams
  - Other NLP applications of log frequency BF framework

Some References

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Thanks

- Thanks for listening!
- Thanks to all the Moses Team!